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MATHEMATICAL MODEL OF THE SPREAD OF FOOT AND MOUTH DISEASES (FMD)

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ABSTRACT
Foot and Mouth Disease (FMD) is a contagious animal disease and can cause death. FMD is caused by an RNA virus belonging to the genus Apthovirus, family Picornaviridae. FMD can be transmitted through direct contact with exposed animals, indirect contact and through the air. In this research, a mathematical model of the spread of FMD in animal populations will be constructed by adapting the SIR (Susceptible, Infected, Recovered) model. From this model two critical points are obtained. The first critical point(TK_1) is the disease-free critical point and the second critical point (TK_2) is the endemic critical point for FMD. The existence of TK_1 can be guaranteed, because all parameters are positive. And it is stable jika $\beta < \frac{(\mu+\delta+\omega+\gamma_1)\mu(\mu+\varphi+\sigma)}{A(\mu+\varphi)}$. Furthermore TK_2 exists and is stable if $\beta > \frac{(\mu^2+(\varphi+\omega+\sigma)\mu+\omega\sigma)(\mu+\delta+\omega+\gamma_1)}{(\mu+\omega+\varphi)A}$.

Keywords : Mathematical Models, FMD, Existence, Stability

I. INTRODUCTION

In 2022, Indonesia has declared foot and mouth disease (FMD) an epidemic with the Decree of the Minister of Agriculture Number 500.1/KTPS/PK.300/M/06/2022 (Kementan, 2022). FMD is caused by RNA belonging to the genus Apthovirus, family Picornaviridae. The FMD virus consists of 7 serotypes, namely O, A, C, Southern African Territories (SAT-1, SAT-2 and SAT-3) and Asia-1 (Kementan, 2022).

Based on history, in 1887 FMD disease was introduced through the importation of dairy cattle from the Netherlands, and there were several outbreaks. Furthermore, in 1983 the last FMD outbreak was in Java. Eradication with mass vaccination. So in 1986 the national declaration of Indonesia's FMD-free status was issued with the issuance of Minister of Agriculture Decree NO. 260/Kpts/TN.510/5/1986. Then in 1990 the recognition of FMD-free status in Indonesia by the World Animal Health Organization (OIE), was stated in OIE resolution No. XI in 1990 (Kementan, 2022).

This FMD attacks cloven-hoofed animals such as cows, buffalo, pigs, sheep and goats. This disease does not harm human health. Meat and milk are still safe to consume as long as they are cooked properly. However, losses caused by FMD include a decrease in animal milk production, sudden death, miscarriage, infertility, weight loss, and barriers to animal trade (export). This disease is transmitted to other animals in 3 ways, namely direct contact, indirect contact, and airborne spread. And there have been many ways to break the chain of spread of FMD, in the form of quarantining infected animals, eliminating sources of infection by destroying limited animals, both infected and exposed, decontaminating cages, equipment or other things that can transmit this disease, and provide vaccinations for susceptible animals (Kementan, 2022).

There has been a lot of research conducted regarding FMD. Nur w, et al (2023) with title "Mathematical Model of Foot And Mouth Disease Considering Vaccination Disinfection And Early Quarantine", The results of this research show that vaccination and quarantine can control the spread of FMD. Fahrudin I, et al (2023) with the title "Mathematical Modeling of Foot And Mouth Disease Spread On Livestock Using Saturated Incidence Rate". The results of his research show that the parameters that greatly influence the spread of FMD are direct or indirect contact (which causes the introduction of the FMD virus) and the carrying capacity of livestock. Then the most influential parameter in reducing the spread of FMD is the application of culling to exposed livestock and infected livestock.

Based on the background above, the author is also interested in modeling the phenomenon of the spread of FMD in Palu City. Because mathematical modeling is the only instrument that allows fast, accurate, cheap and safe endemic analysis. The constructed mathematical model of epidemic involves animal and human populations as vectors. Next, a dynamic analysis will be carried out on the stability of the equilibrium point which describes the endemic conditions of FMD. The next stage is to create a simulation of the model that has been built using Maple software which describes the endemic conditions of FMD. This research is expected to be useful for government programs in controlling FMD.

II. METHODS

The mathematical model of the epidemic that will be constructed refers to the SIR model (*Susceptible*, *Infected, Recovered*), In this study, the animal population was divided into four subpopulations, namely the susceptible animal subpopulation (S), subpopulation of animals infected with FMD (I), The subpopulation of animals infected with FMD is quarantined and a subpopulation of animals that recovered from FMD (R). Analysis of the model is carried out by determining critical points and the stability of these critical points using the linearization method. Remembering that the model formed is a non-linear system. In determining critical points, various algebraic manipulations need to be carried out to check the existence conditions for each critical point. This is because the population number cannot be negative. For example, given a system of differential equations, as follows:

$$\frac{dX(t)}{dt} = F(X(t)) \tag{1}$$

Where $X(t) = \begin{pmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{pmatrix}$ where m represents the number of differential equations. Titik-titik kritis dari sistem, ${x_i}^*$ for $i = 1, \dots, n,$ with n representing the number of critical points, can be obtained by reviewing the system in a stagnant condition, (Perko, 1991) that is

$$F({X_i}^*) = 0 \tag{2}$$

Linearization around the critical point will produce a Jacobian matrix, with the following formulation:

$$J({X_i}^*) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}({X_i}^*) & \dots & \frac{\partial f_1}{\partial x_m}({X_i}^*) \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1}({X_i}^*) & \dots & \frac{\partial f_m}{\partial x_m}({X_i}^*) \end{bmatrix}, i = 1, \dots, n; j = 1, \dots, m \tag{3}$$

where $f_j = F(x_j)$. Furthermore, the stability properties of the critical point can be determined by analyzing the sign of the eigenvalues obtained from the Jacobian matrix. If the real part of all eigenvalues is negative then the critical point will be locally aismtotic stable. However, if there is at least one eigenvalue with a positive real part then the critical point will be unstable (Olsder and Woude, 1994). In this way, a condition can be identified that determines the stability properties of a critical point. If a critical point is stable then the solutions around it will converge to the critical point, and vice versa. In this way, the behavior of the model solution in the future can be known if the conditions at a certain time are known.

III. RESULTS AND DISSCUSSION

3.1. Mathematical Model

The mathematical model of the epidemic that will be constructed refers to the SIR model (*Susceptible*, *Infected, Recovered*), In this study, the animal population was divided into four subpopulations, namely the susceptible animal subpopulation(S), subpopulation of animals infected with FMD (I), The subpopulation of animals infected with FMD is quarantined (K) and a subpopulation of animals recovered from FMD (R). The transmission diagram of this model can be seen in the Figure 1.

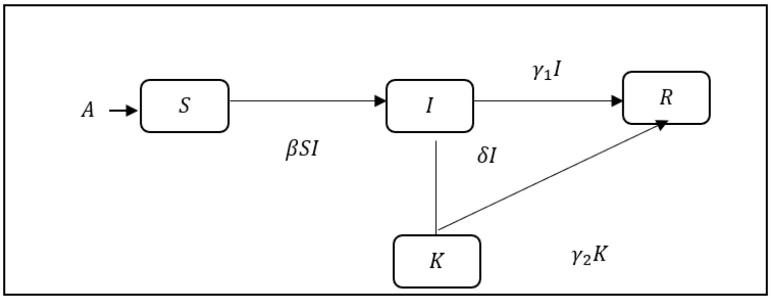

Figure 1 : Diagram of the Distribution of Foot and Mouth Disease (FMD)

Table 1. Parameter Values

Parameter	Definition	Value	Reference
A	Recruitment rate of susceptible animal subpopulations	$1\ day^{-1}$	Nur w et al (2023)
β	Rate of FMD infection by subpopulations of FMD-infected animals	$0.33\ day^{-1}$	Tedesse et al (2019)
φ	The rate of movement of a subpopulation of recovered animals to a subpopulation of susceptible animals	$\frac{1}{6x4x7}\ day^{-1}$	Ringa and Bauch (2014)
σ	Vaccination rate of susceptible animal subpopulations	$0.54\ day^{-1}$	Sari A N (2023)
δ	The rate of movement of a subpopulation of infected animals to a subpopulation of quarantine animals	$\frac{1}{7}\ day^{-1}$	Nur w et al (2023)
γ_1	Recovery rate of subpopulations of infected animals	$0.143\ day^{-1}$	Ringa and Bauch (2014)
γ_2	Recovery rate of quarantine animal subpopulations	$0.143\ day^{-1}$	Ringa and Bauch (2014)
μ	Natural death rate in animal subpopulations	$\frac{1}{365x1.5}\ day^{-1}$	Nur w et al (2023)
ω	Animal death rate due to FMD virus	$0.01\ day^{-1}$	Sari A N 2023

Based on the transmission diagram in Figure 1, the FMD transmission model can be written into the following system of differential equations.

$$
\begin{aligned}
\frac{dS}{dt} &= A + \varphi R - \beta SI - \sigma S - \mu S \\
\frac{dI}{dt} &= \beta SI - \gamma_1 I - \varphi I - (\mu + \omega) I \\
\frac{dK}{dt} &= \varphi I - \gamma_2 K - (\mu + \omega) K \\
\frac{dR}{dt} &= \gamma_1 I + \gamma_2 K + \sigma S - \varphi R - \mu R
\end{aligned} \qquad (4)
$$

3.2. The Existence of a Tipping Point

From the model two critical points are obtained, where the first critical point describes a disease-free condition. Meanwhile, the second critical point describes an endemic critical point. The critical points are as follows:

$$TK_1 = \left(\frac{A(\mu+\varphi)}{\mu(\mu+\varphi+\sigma)}, 0, 0, \frac{A\sigma}{\mu(\mu+\varphi+\sigma)}\right)$$

$$TK_2 = (S^*, I^*, K^*, R^*, M^*)$$

where

$$S^* = \frac{\delta + \mu + \omega + \gamma_1}{\beta}$$

$$I^* = \frac{(\mu+\omega+\gamma_2)\begin{pmatrix} -\mu^3 + (-\varphi-\delta-\omega-\sigma-\gamma_1)\mu^2 \\ +\big((-\varphi-\sigma)\omega + (-\delta-\gamma_1)\varphi + A\beta - \delta\sigma - \sigma\gamma_1\big)\mu + A\beta\varphi \end{pmatrix}}{\left(\beta\begin{pmatrix} \mu^3 + (\varphi+\delta+2\omega+\gamma_1+\gamma_2)\mu^2 \\ +\big(\omega^2 + (2\varphi+\delta+\gamma_1+\gamma_2)\omega + (\varphi+\gamma_2)\varphi + \gamma_2(\delta+\gamma_1)\big)\mu + \varphi\omega(\omega+\delta+\gamma_2) \end{pmatrix}\right)}$$

$$K^* = \frac{\Big(-\mu^3 + (-\varphi-\delta-\omega-\sigma-\gamma_1)\mu^2 + \big((-\varphi-\sigma)\omega + (-\delta-\gamma_1)\varphi + A\beta - \delta\sigma - \sigma\gamma_1\big)\mu + A\beta\varphi\Big)\delta}{\left(\beta\begin{pmatrix} \mu^3 + (\varphi+\delta+2\omega+\gamma_1+\gamma_2)\mu^2 \\ +\big(\omega^2 + (2\varphi+\delta+\gamma_1+\gamma_2)\omega + (\delta+\gamma_2)\varphi + \gamma_2(\delta+\gamma_1)\big)\mu + \varphi\omega(\omega+\delta+\gamma_2) \end{pmatrix}\right)}$$

$$R^* = \frac{\begin{pmatrix} (\sigma-\gamma_1)\mu^3 + \big((3\sigma-2\gamma_1)\omega - {\gamma_1}^2 + (-\delta+\sigma-\gamma_2)\gamma_1 + (-\delta+\sigma)\gamma_2 + 2\delta\sigma\big)\mu^2 + \\ \begin{pmatrix} (3\sigma-\gamma_1)\omega^2 + (-{\gamma_1}^2 + (-\delta+2\sigma-\gamma_2)\gamma_1 + (-\delta+2\sigma)\gamma_2 + 4\delta\sigma)\omega - \gamma_2{\gamma_1}^2 \\ +\big((-2\delta+\sigma)\gamma_2 + A\beta + \delta\sigma\big)\gamma_1 + \delta\big((-\delta+\sigma)\gamma_2 + \delta\sigma\big) \end{pmatrix}\mu + \omega^3\sigma \\ +2\sigma\left(\delta + \frac{1}{2}\gamma_1 + \frac{1}{2}\gamma_2\right)\omega^2 + \big((A\beta+\delta\sigma+\sigma\gamma_2)\gamma_1 + \delta\sigma(\delta+\gamma_2)\big)\omega + A\beta\gamma_2(\delta+\gamma_1) \end{pmatrix}}{\left(\beta\begin{pmatrix} \mu^3 + (\varphi+\delta+2\omega+\gamma_1+\gamma_2)\mu^2 + \\ +(\omega^2 + (2\varphi+\delta+\gamma_1+\gamma_2)\omega + \gamma_1\gamma_2 + (\varphi+\delta)\gamma_2 + \varphi\delta)\mu + \varphi\omega(\omega+\delta+\gamma_2) \end{pmatrix}\right)}$$

The existence of TK_1 can be guaranteed because all parameter values are positive. Meanwhile, for TK_2, its existence can be guaranteed with conditions

$$\beta > \frac{(\mu^2 + (\varphi+\omega+\sigma)\mu + \omega\sigma)(\mu+\delta+\omega+\gamma_1)}{(\mu+\omega+\varphi)A}$$

3.3. Critical Point Stability Analysis

The stability of the critical point is determined based on the eigenvalue λ obtained from the characteristic polynomial

$$P(\lambda) = \det(J(TK) - \lambda I) = 0, \qquad (7)$$

Where $J(TK)$ is the Jacobian matrix evaluated at the critical point. From TK_1, the following characteristic polynomial can be obtained.

$$P(\lambda) = -\frac{1}{\mu(\mu+\varphi+\sigma)}\Big((A\beta\mu + A\beta\varphi - \delta\mu^2 - \delta\mu\varphi - \delta\mu\sigma - \lambda\mu^2 - \lambda\mu\varphi - \lambda\mu\sigma - \mu^3 - \mu^2\omega - \mu^2\varphi - \mu^2\sigma - \mu^2\gamma_1 - \mu\omega\varphi - \mu\omega\sigma - \mu\varphi\gamma_1 - \mu\sigma\gamma_1)(\mu+\omega+\gamma_2+\lambda)(\lambda^2 + (2\mu+\varphi+\sigma)\lambda + \mu^2 + \mu\varphi + \mu\sigma)\Big)$$

Based on the characteristic polynomial $P(\lambda)$, it is known that this polynomial has 3 negative roots, namely, $\lambda_1 = -\mu - \omega - \gamma_2, \lambda_2 = -\mu$u$, \lambda_3 = -\mu - \varphi - \sigma$ and $\lambda_4 = \frac{-\mu^3 + (-\varphi-\delta-\omega-\sigma-\gamma_1)\mu^2 + ((-\delta-\omega-\gamma_1)\varphi + (-\delta-\omega-\gamma_1)\sigma + A\beta)\mu + A\beta\varphi}{\mu(\mu+\varphi+\sigma)}$ will be negative if $\beta < \frac{(\mu+\delta+\omega+\gamma_1)\mu(\mu+\varphi+\sigma)}{A(\mu+\varphi)}$

For TK_2, the following characteristic polynomial can be obtained.

$$P(\lambda) = (c_0\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4)$$

where

$c_0 = 1$

$$c_1 = \frac{\left(\begin{array}{c} \omega^4 + (2\gamma_2 + 5\mu + 2\varphi + \delta + \gamma_1)\omega^3 + \left(\begin{array}{c} 9\mu^2 + (7\gamma_2 + 7\varphi + 4\delta + 4\gamma_1)\mu + {\gamma_2}^2 \\ +(3\varphi + 2\delta + 2\gamma_1)\gamma_2 + \varphi^2 + (2\delta + \sigma + \gamma_1)\varphi + A\beta \end{array}\right)\omega^2 \\ + \left(\begin{array}{c} 7\mu^3 + (8\gamma_2 + 8\varphi + 5\delta + 5\gamma_1)\mu^2 + \left(\begin{array}{c} 2{\gamma_2}^2 + (7\varphi + 5\delta + 5\gamma_1)\gamma_2 \\ +2\varphi^2 + (5\delta + 2\sigma + \gamma_1)\varphi + 2A\beta \end{array}\right)\mu \\ +(\varphi + \delta + \gamma_1){\gamma_2}^2 + (\varphi^2 + (2\delta + \sigma + \gamma_1)\varphi + A\beta)\gamma_2 + \varphi(A\beta + \delta\varphi + \delta\sigma) \end{array}\right)\omega + 2\mu^4 \\ +(3\gamma_2 + 3\varphi + 2\delta + 2\gamma_1)\mu^3 + ({\gamma_2}^2 + (4\varphi + 3\delta + 3\gamma_1)\gamma_2 + \varphi^2 + (3\delta + \sigma)\varphi + A\beta)\mu^2 \\ +\big((\varphi + \delta + \gamma_1){\gamma_2}^2 + (\varphi^2 + (\delta + \sigma)\varphi + A\beta)\gamma_2 + \varphi(A\beta + \delta\varphi + \delta\sigma)\big)\mu + A\beta\varphi\gamma_2 \end{array}\right)}{\big((\omega+\mu)(\omega^2 + (\gamma_2 + 2\mu + \varphi + \delta + \gamma_1)\omega + \mu^2 + (\gamma_2 + \varphi + \delta + \gamma_1)\mu + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)\big)}$$

$$c_2 = \frac{\left(\begin{array}{c} (\gamma_2 + \varphi - \delta - \sigma - \gamma_1)\mu^4 + \left(\begin{array}{c} (3\gamma_2 + 4\varphi - 3\delta - 4\sigma - 3\gamma_1)\omega + {\gamma_2}^2 + (3\varphi - \sigma)\gamma_2 + \varphi^2 \\ +(\sigma - 3\gamma_1)\varphi - \delta^2 + (-2\sigma - 2\gamma_1)\delta + 3A\beta - 2\sigma\gamma_1 - {\gamma_1}^2 \end{array}\right)\mu^3 \\ + \left(\begin{array}{c} (3\gamma_2 + 6\varphi - 3\delta - 6\sigma - 3\gamma_1)\omega^2 + \left(\begin{array}{c} 2{\gamma_2}^2 + (8\varphi - 3\sigma)\gamma_2 + 3\varphi^2 \\ +(\delta + 2\sigma - 4\gamma_1)\varphi - 2\delta^2 + (-6 - 4\gamma_1)\delta \\ +8A\beta - 6\sigma\gamma_1 - 2{\gamma_1}^2 \end{array}\right)\omega \\ +(2\varphi + \delta + \gamma_1){\gamma_2}^2 + \left(\begin{array}{c} 2\varphi^2 + (-\delta + 2\sigma - 3\gamma_1)\varphi - \delta^2 \\ +(-2\sigma - 2\gamma_1)\delta + 4A\beta - 2\sigma\gamma_1 - {\gamma_1}^2 \end{array}\right)\gamma_2 + (\delta - \gamma_1)\varphi^2 \\ +(-\delta^2 + (\sigma - 2\gamma_1)\delta + 4A\beta - \sigma\gamma_1 - {\gamma_1}^2)\varphi + (\delta + \gamma_1) \\ (A\beta - \delta\sigma - \sigma\gamma_1) \end{array}\right)\mu^2 \\ + \left(\begin{array}{c} (\gamma_2 + 4\varphi - \delta - 4\sigma - \gamma_1)\omega^3 + \left(\begin{array}{c} {\gamma_2}^2 + (7\varphi - 3\sigma)\gamma_2 + 3\varphi^2 + (2\delta + \sigma)\varphi - \delta^2 \\ +(-6\sigma - 2\gamma_1)\delta + 7A\beta - 6\sigma\gamma_1 - {\gamma_1}^2 \end{array}\right)\omega^2 \\ + \left(\begin{array}{c} (3\varphi + \delta + \gamma_1){\gamma_2}^2 + \left(\begin{array}{c} 4\varphi^2 + (2\delta + 3\sigma)\varphi - \delta^2 \\ +(-4\sigma - 2\gamma_1)\delta + 7A\beta - 4\sigma\gamma_1 - {\gamma_1}^2 \end{array}\right)\gamma_2 \\ +(2\delta - \gamma_1)\varphi^2 + (-\delta^2 + (\sigma - 2\gamma_1)\delta + 7A\beta - 2\sigma\gamma_1 - {\gamma_1}^2)\varphi + 2(\delta + \gamma_1) \\ (A\beta - \delta\sigma - \sigma\gamma_1) \end{array}\right)\omega \\ +(A\beta + \varphi^2 + \varphi\sigma){\gamma_2}^2 + \left(\begin{array}{c} -\varphi^2\gamma_1 + (5A\beta - \delta^2 - 2\delta\gamma_1 - \sigma\gamma_1 - {\gamma_1}^2)\varphi \\ +(\delta + \gamma_1)(A\beta - \delta\sigma - \sigma\gamma_1) \end{array}\right)\gamma_2 \\ +A\varphi\beta(\varphi + \delta + \gamma_1) \end{array}\right)\mu \\ +(\omega + \gamma_2)\left(\begin{array}{c} (\varphi - \sigma)\omega^3 + (\gamma_2\varphi + \varphi^2 + (\delta + \gamma_1)\varphi + 2A\beta - 2\delta\sigma - 2\sigma\gamma_1)\omega^2 \\ + \left(\begin{array}{c} (\varphi^2 + (\delta + \sigma + \gamma_1)\varphi + A\beta)\gamma_2 \\ +\varphi^2\delta + (3A\beta - \sigma\gamma_1)\varphi + (\delta + \gamma_1)(A\beta - \delta\sigma - \sigma\gamma_1) \end{array}\right)\omega \\ +A\varphi\beta(\gamma_2 + \varphi + \delta + \gamma_1) \end{array}\right) \end{array}\right)}{\big((\omega+\mu)(\mu^2 + (\gamma_2 + \varphi + \delta + 2\omega + \gamma_1)\mu + \omega^2 + (\gamma_2 + \varphi + \delta + \gamma_1)\omega + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)\big)}$$

$$
c_3 = \frac{\left(2(\mu+\omega+\gamma_2)\left(\begin{array}{c}
-\frac{2}{3}\mu^5 + \left(-\frac{7}{3}\omega - \frac{1}{3}\gamma_2 - \frac{2}{3}\sigma - \frac{4}{3}\gamma_1 - \varphi - \frac{4}{3}\delta\right)\mu^4 \\
+\left(\begin{array}{c}
-3\omega^2 + \left(-\gamma_2 - \frac{10}{3}\gamma_1 - \frac{10}{3}\delta - \frac{7}{3}\sigma - \frac{8}{3}\varphi\right)\omega \\
+\left(-\frac{2}{3}\gamma_1 - \frac{2}{3}\delta - \frac{1}{3}\sigma - \frac{1}{3}\varphi\right)\gamma_2 - \frac{1}{3}\varphi^2 \\
+\left(-2\gamma_1 - \frac{1}{3}\sigma - 2\delta\right)\varphi \\
-\frac{2}{3}\delta^2 + \left(-\frac{4}{3}\sigma - \frac{4}{3}\gamma_1\right)\delta - \frac{2}{3}{\gamma_1}^2 - \frac{4}{3}\sigma\gamma_1 + A\beta
\end{array}\right)\mu^3 \\
+\left(\begin{array}{c}
-\frac{5}{2}\omega^3 + \left(-\frac{8}{3}\gamma_1 - 3\sigma - \gamma_2 - \frac{7}{3}\varphi - \frac{8}{3}\delta\right)\omega^2 \\
+\left(\begin{array}{c}
\left(-\sigma - \frac{4}{3}\delta - \frac{4}{3}\gamma_1 - \frac{2}{3}\varphi\right)\gamma_2 - \frac{2}{3}\varphi^2 \\
+\left(-\frac{10}{3}\delta - \frac{4}{3}\sigma - 3\gamma_1\right)\varphi \\
-\delta^2 + \left(-2\gamma_1 - \frac{10}{3}\sigma\right)\delta \\
-\frac{10}{3}\sigma\gamma_1 - {\gamma_1}^2 + \frac{7}{2}A\beta
\end{array}\right)\omega \\
+\left(\begin{array}{c}
(-\delta-\gamma_1)\varphi - \frac{1}{3}\delta^2 \\
+\left(-\frac{2}{3}\gamma_1 - \frac{2}{3}\sigma\right)\delta + \frac{2}{3}A\beta - \frac{2}{3}\sigma\gamma_1 - \frac{1}{3}{\gamma_1}^2
\end{array}\right)\gamma_2 \\
+\left(-\frac{2}{3}\delta - \frac{2}{3}\gamma_1\right)\varphi^2 + \left(\begin{array}{c}
-\delta^2 + \left(-\frac{2}{3}\sigma - \frac{5}{3}\gamma_1\right)\delta \\
+\frac{5}{3}A\beta - \frac{2}{3}\sigma\gamma_1 - \frac{2}{3}{\gamma_1}^2
\end{array}\right)\varphi \\
+\frac{2}{3}(\delta+\gamma_1)(A\beta - \delta\sigma - \sigma\gamma_1)
\end{array}\right)\mu^2 \\
+\left(\begin{array}{c}
-\frac{1}{3}\omega^4 + \left(\begin{array}{c}
-\frac{1}{3}\gamma_2 - \frac{5}{3}\sigma \\
-\frac{2}{3}\gamma_1 - \frac{2}{3}\varphi - \frac{2}{3}\delta
\end{array}\right)\omega^3 \\
+\left(\begin{array}{c}
\left(-\frac{2}{3}\gamma_1 - \sigma - \frac{1}{3}\varphi - \frac{2}{3}\delta\right)\gamma_2 \\
-\frac{1}{3}\varphi^2 + \left(-\frac{5}{3}\sigma - \gamma_1 - \frac{4}{3}\delta\right)\varphi \\
-\frac{1}{3}\delta^2 + \left(-\frac{2}{3}\gamma_1 - \frac{8}{3}\sigma\right)\delta \\
-\frac{8}{3}\sigma\gamma_1 + \frac{5}{3}A\beta - \frac{1}{3}{\gamma_1}^2
\end{array}\right)\omega^2 \\
+\left(\begin{array}{c}
\left(\begin{array}{c}
\left(-\frac{2}{3}\gamma_1 - \frac{2}{3}\delta - \frac{1}{3}\sigma\right)\varphi \\
-\frac{1}{3}\delta^2 + \left(-\frac{4}{3}\sigma - \frac{2}{3}\gamma_1\right)\delta \\
-\frac{1}{3}{\gamma_1}^2 - \frac{4}{3}\sigma\gamma_1 + A\beta
\end{array}\right)\gamma_2 \\
+\left(-\frac{2}{3}\delta - \frac{2}{3}\gamma_1\right)\varphi^2 + \left(\begin{array}{c}
-\frac{2}{3}\delta^2 + \\
\left(-\frac{5}{3}\sigma - \gamma_1\right)\delta \\
+\frac{8}{3}A\beta - \frac{1}{3}{\gamma_1}^2 - \frac{4}{3}\sigma\gamma_1
\end{array}\right)\varphi \\
+(\delta+\gamma_1)(A\beta - \delta\sigma - \sigma\gamma_1)
\end{array}\right)\omega \\
+\left(\begin{array}{c}
\left(-\frac{1}{3}\delta - \frac{1}{3}\gamma_1\right)\varphi^2 + \left(\begin{array}{c}
-\frac{1}{3}\delta^2 + \\
\left(-\frac{2}{3}\gamma_1 - \frac{1}{3}\sigma\right)\delta \\
-\frac{1}{3}{\gamma_1}^2 + A\beta - \frac{1}{3}\sigma\gamma_1
\end{array}\right)\varphi \\
\frac{1}{3}(\delta+\gamma_1)(A\beta - \delta\sigma - \sigma\gamma_1)
\end{array}\right)\gamma_2 \\
+\frac{2}{3}\varphi\left(\begin{array}{c}
\left(-\frac{1}{2}\gamma_1\delta - \frac{1}{2}\delta^2 + A\beta\right)\varphi - \frac{1}{2}\delta^2\sigma \\
+\left(\frac{3}{2}A\beta - \frac{1}{2}\sigma\gamma_1\right)\delta + A\beta\gamma_1
\end{array}\right)
\end{array}\right)\mu \\
+\frac{1}{3}\left(-\omega^2\sigma + (A\beta - \delta\sigma - \sigma\gamma_1)\omega + A\beta\varphi\right) \\
(\omega^2 + (\gamma_2 + 2\varphi + \delta + \gamma_1)\omega + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)
\end{array}\right)\right)}{\left((\omega+\mu)(\mu^2 + (\gamma_2 + \varphi + \delta + 2\omega + \gamma_1)\mu + \omega^2 + (\gamma_2 + \varphi + \delta + \gamma_1)\omega + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)\right)}
$$

$$c_4 = \frac{\left(\left(\begin{array}{c}-\mu^3 + (-\varphi - \delta - 2\omega - \sigma - \gamma_1)\mu^2 \\ +(-\omega^2 + (-\varphi - \delta - 2\sigma - \gamma_1)\omega + (-\delta - \gamma_1)\varphi + A\beta - \sigma\delta - \sigma\gamma_1)\mu \\ -\omega^2\sigma + (A\beta - \delta\sigma - \sigma\gamma_1)\omega + A\beta\varphi\end{array}\right) (\mu + \omega + \gamma_2)\left(\begin{array}{c}\mu^3 + (\gamma_2 + \varphi + \delta + 2\omega + \gamma_1)\mu^2 \\ +(\omega^2 + (\gamma_2 + 2\varphi + \delta + \gamma_1)\omega + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)\mu \\ +\varphi\omega(\omega + \delta + \gamma_2)\end{array}\right)\right)}{\left((\omega + \mu)(\mu^2 + (\gamma_2 + \varphi + \delta + 2\omega + \gamma_1)\mu + \omega^2 + (\gamma_2 + \varphi + \delta + \gamma_1)\omega + (\varphi + \delta + \gamma_1)\gamma_2 + \varphi\delta)\right)}$$

Based on the characteristic polynomial P(λ), it is known that this polynomial has a complicated formula making it difficult to determine its value. Using Descartes' rule, so that the fourth order polynomial of $P(\lambda)$ has roots with negative real parts where $c_0 > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 > 0$ and $c_4 > 0$. So that's the condition $\beta > \frac{(\mu^2+(\varphi+\omega+\sigma)\mu+\omega\sigma)(\mu+\delta+\omega+\gamma_1)}{(\mu+\omega+\varphi)A}$.

3.4. Numerical Simulation

In this section, numerical simulations of the FMD distribution model will be presented to support the analytical results. The simulation results use initial values of $(S, I, K, R) = (87,3,3,5)$ and the parameter values in Table 1 can be seen in Figure 2. The figure shows that the number of subpopulations of infected animals is quarantined. Initially it shows an increase and in the end it decreases until 35 days and will reach the critical point of coexistence. The increase that occurred was due to the rate of infection of a subpopulation of susceptible animals. Because it reduces the subpopulation of infected and quarantined animals. Causing a considerable increase in the subpopulation of animals recovered for 60 days and will reach the critical point of their coexistence.

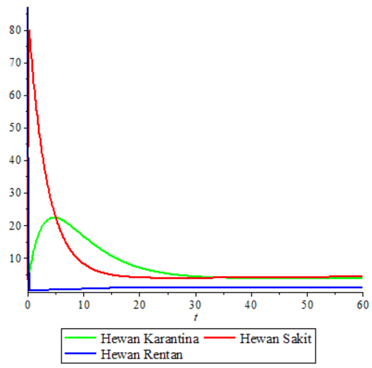

Figure (2)

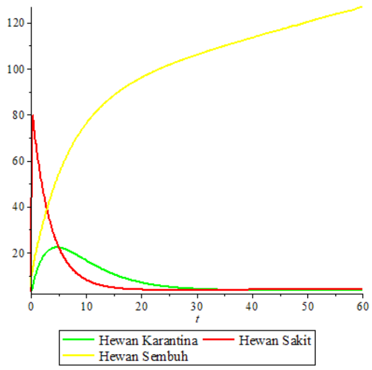

Figure (3)

IV. CONCLUSION

1. The mathematical model of foot and mouth disease (FMD) is as follows:

$$\frac{dS}{dt} = A + \varphi R - \beta SI - \sigma S - \mu S$$
$$\frac{dI}{dt} = \beta SI - \gamma_1 I - \varphi I - (\mu + \omega)I$$
$$\frac{dK}{dt} = \varphi I - \gamma_2 K - (\mu + \omega)K$$
$$\frac{dR}{dt} = \gamma_1 I + \gamma_2 K + \sigma S - \varphi R - \mu R$$

2. From the mathematical model of foot and mouth disease (FMD), two critical points (TK) are obtained(TK). (TK_1) is a disease-free critical point that exists unconditionally and is stable if $\beta < \frac{(\mu+\delta+\omega+\gamma_1)\mu(\mu+\varphi+\sigma)}{A(\mu+\varphi)}$. Meanwhile, (TK_2) is the critical point where endemics exist and are stable if $\beta > \frac{(\mu^2+(\varphi+\omega+\sigma)\mu+\omega\sigma)(\mu+\delta+\omega+\gamma_1)}{(\mu+\omega+\varphi)A}$.

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