Jurnal Ilmiah Matematika dan Terapan
Volume 21 Number 1 June 2024 (Pages 46 - 53)
doi: https://doi.org/10.22487/2540766X.2024.v21.i1.17082

Jurnal Ilmiah Matematika dan Terapan
ISSN : 2540 – 766X

APPLICATION OF MAX-PLUS ALGEBRA IN DETERMINING THE SHORTEST ROUTE OF GOODS DISTRIBUTION ON JALUR NUGRAHA EKAKURIR (JNE) IN PALU CITY

Andri[1], Desy Lusiyanti[2], and Selvy Musdalifah[3]

[1,2,3]Study Program of Mathematics, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Tadulako University
[1]andri.math2007@gmail.com, [2]desylusiyanti@gmail.com, [3]selvymusdalifah@gmail.com
*(*corresponding author)*

ABSTRACT

Determining the shortest route is a solution that is needed for companies engaged in the distribution of goods, because the shortest route can help companies optimize the distance traveled and streamline the time needed. Therefore, this study aims to apply max-plus algebra to determine the shortest route for the distribution of goods on the Jalur Nugraha Ekakurir (JNE) in Palu City. This method was chosen because Max-Plus Algebra can find more optimal results from the matrix exponentiation operation of a weighted graph. The data used were obtained from previous research which consisted of 13 JNE warehouse points along with the distance between these points. The results to be obtained are in the form of the shortest route between one point and another which is represented in a graph path with each path length and weight obtained based on the results of max-plus algebraic calculations. Of all the possible route, obtained the route with the minimum weight for distribution of goods from JNE main warehouse, Dewi Sartika *St.* (v_1) to JNE, UNTAD 1 (v_{13}), that is JNE main warehouse, Dewi Sartika *St.* (v_1) → to JNE, Basuki Rahmat *St.* (v_4) → JNE, Sisingamangaraja *St.* (v_8) → JNE, UNTAD 1 (v_{13}) with a total distance of 13.3 km.

Keywords : Max-plus algebra, Graph, JNE, Shortest path

I. INTRODUCTION

From the official website page, Jalur Nugraha Ekakurir (JNE) is a company engaged in shipping and logistics with the official name PT. Tiki Jalur Nugraha Ekakurir was founded by H. Soeprapto Soeparno On November 26, 1990, JNE commenced its operations, focusing primarily on managing customs activities and the import of goods and documents, as well as their delivery from international locations to Indonesia. Over time, JNE has also expanded its network at the domestic level by establishing agents spread throughout Indonesia.

Determining the shortest route is a solution that is needed for companies engaged in the distribution of goods, because the shortest route can help companies optimize the distance traveled and streamline the time needed. The shortest path problem involves finding a route between two vertices on a weighted graph such that the total sum of the weights of the edges traversed is minimized[3].

In the dynamic world of logistics, speed and efficiency in goods distribution are key to meeting increasing customer demands. JNE, a leader in the courier industry in Indonesia, continues to strive to improve its service performance. One of the biggest challenges faced by JNE, especially in Palu City, is optimizing goods distribution routes amidst complex geographic conditions and varying infrastructure.

JNE's operations in Palu City often encounter scheduling problems and non-optimal delivery routes, causing delays and inaccuracies in delivery times. This also impacts customer satisfaction negatively. To overcome this challenge, the Max-Plus algebra method can be an innovative solution. Max-Plus algebra is a mathematical system that combines maximum and addition operations, enabling more precise and effective analysis of repetitive dynamic systems such as goods distribution networks. Previous research conducted by Suprayitno[4] stated that the Max-Plus algebra procedure can guarantee the correct solution in determining the shortest route. By modeling the distribution system using Max-Plus algebra, it is hoped that an optimal solution can be found, making JNE's goods distribution routes in Palu City more efficient.

This research is expected to make a significant contribution to improving the efficiency and effectiveness of the goods distribution system at JNE. By implementing Max-Plus algebra, JNE can increase customer satisfaction through faster delivery and lower operational costs. Additionally, this research can serve as a reference for other logistics companies in their efforts to optimize goods distribution routes.

II. METHODS

This research aims to apply the max-plus algebra algorithm to the shortest route problem in JNE goods distribution in Palu City. The data used includes qualitative data in the form of 13 JNE warehouse points in Palu City, starting from the main JNE warehouse on Jln. Dewi Sartika to the JNE UNTAD 1 warehouse, as well as quantitative data in the form of routes and distances between points that can be traversed using the Google Maps application. The research steps taken included a literature study, data collection using interview techniques, and field observation.

From interviews with JNE couriers, it was found that the average distance traveled to distribute goods so far is 15.4 km. This is because the determination of distribution routes by couriers is based only on personal assumptions without proper calculations.

The data obtained is first entered into a distance matrix between points with entries according to max-plus algebra. Then operations are carried out according to the max-plus algebra algorithm, and finally, the shortest route is selected based on the distance matrix obtained at the end of the calculation.

A graph is a set of ordered pairs (V, E) is defined with the notation $G = (V, E)$, where $V(G)$ represents the set of vertices (points), and $E(G)$ represents the set of edges (arcs) that connect pairs of vertices [4].

According to Yulianti[4], graphs can be classified based on their orientation, that is an undirected graph is a graph in which the edges do not have any directional orientation and a directed graph is a graph in which the edges have a specific directional orientation.

Max-plus algebra consists of the set $\mathbb{R} \cup \{\varepsilon\}$, where $\mathbb{R}$ represents all real numbers, augmented with the maximum operation $\oplus$ (o-plus) and the addition operation $\otimes$ (o-times). Max-plus algebra is represented as $\mathbb{R}_{max} = (\mathbb{R}_{\varepsilon}, \oplus, \otimes)$, where $\mathbb{R}_{\varepsilon} = \mathbb{R} \cup \{\varepsilon\}$, with $\varepsilon = \{-\infty\}$. For any $x, y \in \mathbb{R}_{max}$, the operations are defined by $x \oplus y = \max(x, y)$, and $x \otimes y = (x + y)$[5]. Operations on max-plus algebra satisfy the commutative, associative, and distributive properties of $\otimes$ to $\oplus$ [2].

According to Subiono[6], the set of $n \times m$ matrices in max-plus algebra is expressed in terms of $\mathbb{R}_{max}^{n \times m}$, where $n, m \in \mathbb{N}$. For a matrix $A \in \mathbb{R}_{max}^{n \times m}$ can be written as $A = (a_{r,s})$, for $r = 1,2,3, \dots, n$ and $s = 1,2,3, \dots, m$. Matrix operations in max-plus algebra are divided into the addition of $A \oplus B$, the multiplication of $A \otimes B$, and the multiplication of the k scalar with matrix A[5].

The steps in determining the shortest path using max-plus algebra are as follows[2]:

1. Form an edge weight matrix $G = (V, E)$. Suppose the matrix formed is a square matrix $R_{n \times n}$.
2. Change the elements of the R matrix to be negative, by multiplying the elements in the R matrix other than the element ε by (-1). This is done because in max-plus algebra to get the solution of the shortest path problem, the maximum value of the negative elements will be used. Suppose the matrix formed at this stage is the matrix R_1.
3. Perform the operation to the power of the matrix R_1 as many as $(n-1)$ times, where n is the size of the matrix R_1. By raising the matrix R_1, we will get the weight of the path with its being the power.
4. Perform the operation on the matrices obtained in the previous step. Suppose the matrix formed at this stage is the matrix R_2, then:

$$R_2 = \left(\left(\left(R_1 \oplus {R_1}^{\otimes 2} \right) \oplus {R_1}^{\otimes 3} \right) \oplus \dots \right) \oplus {R_1}^{\otimes n-1}$$

5. Change back the elements of the matrix R_2 into positive elements, by multiplying the elements of the matrix R_2 other than ε by (-1). Suppose the matrix that is determined at this stage is the matrix R_2^+.

The R_2^+ matrix obtained in the final step is a matrix that expresses the shortest path obtained by power operations in max-plus algebra. The element $(R_2^+)_{rs}$ is the weight of the shortest path from point s to point r. To determine the weight of the shortest path, it can be seen again the results of the power of the matrix R_1, namely $R_1{}^{\otimes m}$. If $(R_2^+)_{rs} = \left(R_1{}^{\otimes m}\right) \times (-1)$, it means that the weight is the weight of the shortest path from point s to point r, with a path length of m.

III. RESULTS AND DISSCUSSION

In this study, data obtained from previous research by Agusman[1], namely 13 JNE warehouse points in Palu City were used.

V_1 = JNE main warehouse, Dewi Sartika *Street*, South Palu
V_2 = JNE, Karanja Lembah *Street* No. 27, Sigi Biromaru
V_3 = JNE, Banteng *Street* No. 8a, South Palu
V_4 = JNE, Basuki Rahmat *Street*, South Palu
V_5 = JNE, Emmy Saelan *Street* No. 20, South Palu
V_6 = JNE, Anoa *Street* No. 107, South Palu
V_7 = JNE, Muhammad Hatta *Street* No. 42, South Palu
V_8 = JNE, Sisingamangaraja *Street*, Mantikulore
V_9 = JNE, Kimaja *Street*, East Palu
V_{10} = JNE, Diponegoro *Street*, Ulujadi
V_{11} = JNE, Tombolotutu *Street*, Mantikulore
V_{12} = JNE, Sam Ratulangi *Street* No. 66a, East Palu
V_{13} = JNE, UNTAD 1, Mantikulore

The following routes may be taken as a route of distribution goods by JNE in Palu city with starting point at the JNE main warehouse on Jalan Dewi *St.* and the ending point at JNE, UNTAD 1.

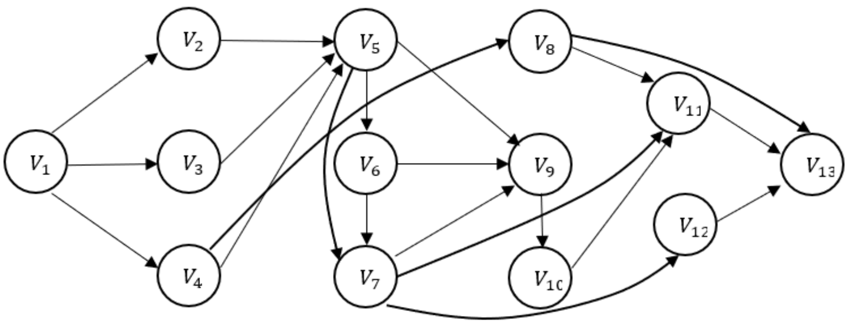

Figure 1 : JNE distribution route in Palu city

To determine the distance between points, google maps is used which is presented in the following table.

Table 1 : Distance between JNE warehouses in Palu city

Counter Asal → Counter Selanjutnya	Jarak (km)
$V_1 \rightarrow V_2$	5,7
$V_1 \rightarrow V_3$	2,2
$V_1 \rightarrow V_4$	3,3
$V_2 \rightarrow V_5$	3,4
$V_3 \rightarrow V_5$	2,9
$V_4 \rightarrow V_5$	2,3
$V_4 \rightarrow V_8$	3,4
$V_5 \rightarrow V_6$	0,9
$V_5 \rightarrow V_7$	2,1
$V_5 \rightarrow V_9$	2,8
$V_6 \rightarrow V_7$	2,7
$V_6 \rightarrow V_9$	2,7
$V_7 \rightarrow V_9$	2,1
$V_7 \rightarrow V_{11}$	2,8
$V_7 \rightarrow V_{12}$	2,1
$V_8 \rightarrow V_{11}$	1,6
$V_8 \rightarrow V_{13}$	6,6
$V_9 \rightarrow V_{10}$	4,3
$V_{10} \rightarrow V_{11}$	3,6
$V_{11} \rightarrow V_{13}$	7,1
$V_{12} \rightarrow V_{13}$	7

The first step to be able to determine the shortest route using max-plus algebra is to build a matrix whose the entries are distances between JNE warehouse. If the matrix is denoted by R, then

$$R = \begin{Bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 5{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2{,}2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 3{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 3{,}4 & 2{,}9 & 2{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0{,}9 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 2{,}1 & 2{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 3{,}4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 2{,}8 & 2{,}7 & 2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 4{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 2{,}8 & 1{,}6 & \varepsilon & 3{,}6 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6{,}6 & \varepsilon & \varepsilon & 7{,}1 & 7 & \varepsilon \end{Bmatrix}$$

Next, multiply all entries in the matrix R other than ε by (-1). Suppose the multiplication result is denoted by R_1, then

$$R_1 = \begin{Bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
-5{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
-2{,}2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
-3{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & -3{,}4 & -2{,}9 & -2{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & -0{,}9 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & -2{,}1 & -2{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & -3{,}4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & -2{,}8 & -2{,}7 & -2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & -4{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & -2{,}8 & -1{,}6 & \varepsilon & -3{,}6 & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & -2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & -6{,}6 & \varepsilon & \varepsilon & -7{,}1 & -7 & \varepsilon
\end{Bmatrix}$$

Perform the operation to the power of the matrix R_1 as many as $(n-1)$ times, so that we get

$$R_1^{\otimes 12} = R_1 \otimes R_1^{\otimes 11} = \begin{Bmatrix}
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon
\end{Bmatrix}$$

Next, the operation $\oplus$ is performed on the matrices that have been obtained in the previous stage. The equation used is as follows.

$$R_2 = \left(\left(\left(\left(\left(\left(\left(\left(\left(\left(R_1 \oplus R_1^{\otimes 2}\right) \oplus R_1^{\otimes 3}\right) \oplus R_1^{\otimes 4}\right) \oplus R_1^{\otimes 5}\right) \oplus R_1^{\otimes 6}\right) \oplus R_1^{\otimes 7}\right) \oplus R_1^{\otimes 8}\right) \oplus R_1^{\otimes 9}\right) \oplus R_1^{\otimes 10}\right) \oplus R_1^{\otimes 11}\right) \oplus R_1^{\otimes 12}$$

Then, will be obtained

$$R_2 = \begin{Bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -5{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -2{,}2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -3{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -5{,}1 & -3{,}4 & -2{,}9 & -2{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -6 & -4{,}3 & -3{,}8 & -3{,}2 & -0{,}9 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -7{,}8 & -5{,}5 & -5 & -4{,}4 & -2{,}1 & -2{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -6{,}7 & \varepsilon & \varepsilon & -3{,}4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -7{,}9 & -6{,}2 & -5{,}7 & -5{,}1 & -2{,}8 & -2{,}7 & -2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -12{,}2 & -10{,}5 & -10 & -9{,}4 & -7{,}1 & -7 & -6{,}4 & \varepsilon & -4{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -8{,}3 & -8{,}3 & -7{,}8 & -5 & -4{,}9 & -5{,}5 & -2{,}8 & -1{,}6 & -7{,}9 & -3{,}6 & \varepsilon & \varepsilon & \varepsilon \\ -9{,}9 & -7{,}6 & -7{,}1 & -6{,}5 & -4{,}2 & -4{,}8 & -2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ -13{,}3 & -14{,}6 & -14{,}1 & -10 & -11{,}2 & -11{,}8 & -9{,}1 & -6{,}6 & -15 & -10{,}7 & -7{,}1 & -7 & \varepsilon \end{Bmatrix}$$

Next, the entries in the R_2 matrix must be positive by multiplying the matrix with (-1). This is done because of problems in the form of mileage. Suppose this product is R_2^+, so that

$$R_2^+ = (-1)R_2 = \begin{Bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 5{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 2{,}2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 3{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 5{,}1 & 3{,}4 & 2{,}9 & 2{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 6 & 4{,}3 & 3{,}8 & 3{,}2 & 0{,}9 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 7{,}8 & 5{,}5 & 5 & 4{,}4 & 2{,}1 & 2{,}7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 6{,}7 & \varepsilon & \varepsilon & 3{,}4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 7{,}9 & 6{,}2 & 5{,}7 & 5{,}1 & 2{,}8 & 2{,}7 & 2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 12{,}2 & 10{,}5 & 10 & 9{,}4 & 7{,}1 & 7 & 6{,}4 & \varepsilon & 4{,}3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 8{,}3 & 8{,}3 & 7{,}8 & 5 & 4{,}9 & 5{,}5 & 2{,}8 & 1{,}6 & 7{,}9 & 3{,}6 & \varepsilon & \varepsilon & \varepsilon \\ 9{,}9 & 7{,}6 & 7{,}1 & 6{,}5 & 4{,}2 & 4{,}8 & 2{,}1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 13{,}3 & 14{,}6 & 14{,}1 & 10 & 11{,}2 & 11{,}8 & 9{,}1 & 6{,}6 & 15 & 10{,}7 & 7{,}1 & 7 & \varepsilon \end{Bmatrix}$$

The R_2^+ matrix contains the minimum weight of the distance that can be traveled in distributing goods on JNE from one point to another. Furthermore, the results of the minimum distance and the shortest path are described in table 2.

Table 2 : The minimum distance and the shortest path of distribution goods on JNE

No	Sisi dari titik v_i ke v_j	Total jarak minumum (km)	Lintasan terpendek	Panjang lintasan
1	(v_1, v_2)	5,7	$v_1 \rightarrow v_2$	1
2	(v_1, v_3)	2,2	$v_1 \rightarrow v_3$	1
3	(v_1, v_4)	3,3	$v_1 \rightarrow v_4$	1
4	(v_1, v_5)	5,1	$v_1 \rightarrow v_3 \rightarrow v_5$	2
5	(v_1, v_6)	6	$v_1 \rightarrow v_3 \rightarrow v_5 \rightarrow v_6$	3
6	(v_1, v_7)	7,8	$v_1 \rightarrow v_3 \rightarrow v_5 \rightarrow v_7$	3
7	(v_1, v_8)	6,7	$v_1 \rightarrow v_4 \rightarrow v_8$	2
8	(v_1, v_9)	7,9	$v_1 \rightarrow v_3 \rightarrow v_5 \rightarrow v_9$	3
9	(v_1, v_{10})	12,2	$v_1 \rightarrow v_3 \rightarrow v_5 \rightarrow v_9 \rightarrow v_{10}$	4
10	(v_1, v_{11})	8,3	$v_1 \rightarrow v_4 \rightarrow v_8 \rightarrow v_{11}$	3

11	(v_1, v_{12})	$9{,}9$	$v_1 \to v_3 \to v_5 \to v_7 \to v_{12}$	4
12	(v_1, v_{13})	$13{,}3$	$v_1 \to v_4 \to v_8 \to v_{13}$	3
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
63	(v_{12}, v_{13})	7	$v_{12} \to v_{13}$	1

From table 2, it can be seen that there are several possibilities that JNE can use in distributing goods from JNE Main warehouse, Dewi Sartika *St.* (v_1) to JNE, UNTAD 1 (v_{13}). For example $(v_1, v_3) \to (v_3, v_5) \to (v_5, v_{11}) \to (v_{11}, v_{13})$, meaning that the distribution of goods will run from the JNE main warehouse of Dewi Sartika *St.* to JNE, Banteng *St.* No. 8a with mileage of $2{,}2$ km. After that the goods are distributed to JNE, Emmy Saelan *St.* no. 20 with a mileage of $2{,}9$ km. Furthermore, the goods are distributed to JNE, Tombolotutu *St.* with a mileage of $4{,}9$ km. Then the goods are distributed to JNE, UNTAD 1 with a distance of $7{,}1$ km. So the total distance traveled for this possibility is $17{,}1$ km.

So from all the possibilities, the minimum distance possible is $(v_1, v_4) \to (v_4, v_8) \to (v_8, v_{13})$ with a total distance of $13{,}3$ km and path length is 4 as shown in table 2 point 12. This means that the distribution of goods move from the JNE main warehouse, Dewi Sartika *St.* to JNE, Basuki rahmat *St.* with a distance of $3{,}3$ km. After that, the goods were distributed to JNE, Sisingamangaraja *St.* with a distance of $3{,}4$ km. Then the goods are distributed to JNE, UNTAD 1 with a distance of $6{,}6$ km, with a total distance of $13{,}3$ km.

By choosing this route, the distribution of goods can be more efficient with a shorter distance of 13.3 km compared to the current 15.4 km used by couriers. This, in turn, will increase delivery speed and reduce delays. With more efficient distribution, JNE can handle more deliveries in the same time period, increasing the overall productivity of JNE operations in Palu.

IV. CONCLUSION

Based on the presented research findings, the following conclusions can be made.

1. Distribution routes used by JNE for delivering goods can be represented as a directed graph. Where each vertex represents JNE warehouses in Palu City, and the edges represent the distance from one warehouse to another.
2. The most efficient route determined through the max-plus algebraic algorithm for distributing goods from JNE main warehouse, Dewi Sartika *St.* to JNE, UNTAD 1 is JNE main warehouse, Dewi Sartika *St.* → to JNE, Basuki Rahmat *St.* → JNE, Sisingamangaraja *St.* → JNE, UNTAD 1 with a total distance of $13{,}3$ km.

The application of the Max-Plus algebra algorithm in the distribution of JNE goods in Palu will bring various significant benefits, including reducing operational costs, increasing delivery speed, enhancing customer satisfaction, and boosting operational productivity. The results of this research will provide a strong basis for JNE to continue improving the efficiency of their goods distribution, ensuring they remain competitive in an increasingly tight logistics market.

Suggestions for further research include combining the Max-Plus algebra algorithm with technologies such as the Internet of Things (IoT) and Big Data Analytics to collect real-time data and monitor route conditions, providing more adaptive and responsive solutions.

REFERENCES

[1]. Dessy. 2011. *Menentukan Lintasan Terpendek dengan Menggunakan Aljabar Max-Plus*. Tesis Matematika. Depok: Universitas Indonesia.

[2]. D. T. Salaki, *Penentuan Lintasan Terpendek Dari Fmipa Ke Rektorat Dan Fakultas Lain Di Unsrat Manado Menggunakan Algoritma Djikstra* J. Ilm. Sains, vol. 11, no. 1, p. 73, 2011, doi: 10.35799/jis.11.1.2011.46.

[3]. Didit Budi Nugroho. 2008. *Pengantar Teori Graf*. Salatiga: Universitas Kristen Satya Wacana.

[4]. H. Suprayitno, *Correctness Proof of Min-plus Algebra for Network Shortest-Paths Simultaneous Calculation,*" vol. 1, no. 1, pp. 14–16, 2017.

[5]. Subiono. 2013. Version 1.1.1. *Aljabar Maxplus dan Terapannya*. Surabaya: FMIPA Institut Teknologi Sepuluh November.