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APPLICATION OF THE COPULA FRANK FOR ESTIMATING VALUE AT RISK (VAR) IN TELECOMMUNICATION SUB SECTOR STOCKS

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ABSTRACT

Investment is investing capital with the aim of getting money or additional profits. When investing, you need to pay attention to risks that can cause losses for investors. One method that is widely used to measure investment risk is Value at Risk (VaR). VaR often has limitations, especially in capturing non-linear dependencies between variables, so a copula function is needed that can handle moderate to strong dependencies. One of the copulas used is the Archimedian copula with Frank subcopula. This article aims to estimate investment risk using the Value at Risk (VaR) method based on the Frank copula approach and to analyze the dependency structure between stock returns. The main steps in estimating VaR using the Frank copula are calculating the return of each stock, estimating the parameters of the Frank copula, carrying out data simulations using Frank copula parameters, calculating the VaR value using Frank copula. The data used in this research comes from shares of PT. Telkom Indonesia Tbk and shares PT. Indosat Ooredoo Hutchison Tbk. These two stocks have a positive correlation of 0.136. However, such a low correlation may still indicate for non-linear dependencies or tail dependencies that cannot be captured by linear correlations, so additional analysis, namely Frank copula, is required. The estimated Frank copula parameter value is 0.825. From the VaR estimation results, the risk obtained at a 90% confidence level is -0.0222, at a 95% confidence level it is -0.0281 and at a 99% confidence level it is -0.0383.

Keywords: *Investment, VaR, Copula, Frank Copula*

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INTRODUCTION

The capital market plays a crucial role in the modern economy, functioning both as a source of financing for companies and as an investment vehicle for the public. The capital market is essentially a place where long-term financial instruments (such as mutual funds, debt securities, stocks, and others) are traded (Islamiyanti et al., 2023). Stocks represent ownership in a company, and their price movements are influenced by multiple factors, including macroeconomic conditions, corporate performance, and global market dynamics. Traditionally, the analysis of relationships between assets has relied on linear correlation measures such as the Pearson correlation coefficient (Solikha, 2012). However, this approach is limited, as it captures only linear relationships and is highly sensitive to non-normal data distributions. To address these shortcomings, Spearman’s Rho correlation is often employed, as it is more robust to variations in data distribution and can capture monotonic, non-linear relationships. Beyond correlation measures, the copula method offers a more flexible statistical framework for modeling dependence structures between variables without assuming identical marginal distributions. Among various copula families, the Frank copula—classified within the Archimedean family—provides a suitable model for capturing moderate and symmetric dependence as well as non-linear associations between variables.

Among the various instruments traded in the capital market, stocks are among the most widely recognized. Stocks represent ownership in a company, and their price movements are influenced by multiple factors, including macroeconomic conditions, corporate performance, and global market dynamics. Traditionally, the analysis of relationships between assets has relied on linear correlation measures such as the Pearson correlation coefficient. However, this approach is limited, as it captures only linear relationships and is highly sensitive to non-normal data distributions. To address these shortcomings, Spearman’s Rho correlation is often employed, as it is more robust to variations in data distribution and can capture monotonic, non-linear relationships. Beyond correlation measures, the copula method offers a more flexible statistical framework for modeling dependence structures between variables without assuming identical marginal distributions. Among various copula families, the Frank copula—classified within the Archimedean family—provides a suitable model for capturing moderate and symmetric dependence as well as non-linear associations between variables.

A number of previous studies have applied the copula approach in Value at Risk (VaR) estimation. Anisha et al., (2021) employed the Frank copula method combined with Monte Carlo simulation to estimate Value at Risk (VaR) for a one-day horizon at a 95% confidence level, obtaining a value of 0.02232676. Similarly, Handini et al., (2018)estimated VaR using the Frank copula–GARCH method with Monte Carlo simulation, reporting VaR values of –0.027883 at the 99% confidence level, –0.01886425 at the 95% confidence level, and –0.01403 at the 90% confidence level.

Research that specifically analyzes the dependency structure and VaR estimation in the Indonesian telecommunications sector is still very limited, even though this sector has its own risk characteristics due to the development of digitalization, market competition, and regulatory dynamics. In addition, previous studies rarely examine how the use of non-parametric dependency measures (Spearman's Rho) combined with Frank copulas affects VaR estimation results in the latest post-pandemic market period.

Building on these prior studies, the present research applies the Frank copula to estimate VaR for Telkom Indonesia and Indosat Ooredoo stocks over the period January 15, 2024, to January 13, 2025. The results of this study are expected to identify and quantify potential risks in investor portfolios, thereby supporting more informed and strategic investment decisions. Furthermore, the findings aim to contribute to the advancement of statistical applications in finance, particularly in the estimation of VaR using the Frank copula with Spearman’s Rho correlation.

MATERIALS AND METHODS

Investments

Investment refers to the allocation of funds with the expectation of generating additional income or profit in the future. In essence, it involves committing a certain amount of capital in the present with the aim of obtaining returns over time. Stock investment, in particular, entails allocating funds for the purchase of shares, with the expectation of earning returns or profits through stock trading activities in the capital market.

Return

Return on shares, which is the reward for investors courage in taking risks on their investments (Shofiyuddin, 2018). The formula for calculating stock returns can be expressed as follows

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (1)$$

where R_t is the stock return in period t, P_t is the investment price at time t, and P_t is the investment price at time $t-1$.

Risk

Risk refers to the uncertainty regarding future outcomes that arise from decisions made under conditions of limited information. In the context of investment, risk can be defined as the degree of deviation between the expected return and the actual return realized (Prowanta, 2019)).

Normality Test

Normality testing is a statistical procedure used to determine whether the distribution of data within a dataset or variable follows a normal distribution. In this study, normality is tested using the Lilliefors corrected Kolmogorov–Smirnov test, where the test statistic is defined asThis test is formulated as follows (Gibbons, 2014)

$$D = |F(z_i) - S(z_i)| \quad (2)$$

Where $F(z_i)$ is is the theoretical probability of values $\leq Z_{hit}(P(Z \leq Z_{hit}))$ and $S(z_i)$ is the empirical cumulative frequency of values $\leq Z_{hit}(P(Z \leq Z_{hit}))$. The decision criteria for the Kolmogorov-Smirnov test are that if the $D < D_{table}$ value or if the $p-value$ is greater than the significance level (α), then the assumption of normality is satisfied. Since the parameters of the normal distribution are estimated from the sample, the Lilliefors correction is applied, and the decision is based on the corresponding p-value.

Autocorrelation Test

Autocorrelation test is the relationship between the residuals of one observation and another. Systematically, the Ljung-Box test is written as follows (Box & Pierce, 1970):

$$Q = n(n+2)\sum_{k=1}^{k} \frac{\rho_k^2}{n-k} \quad (3)$$

Where Q is the Ljung-Box statistics, this statistic follows a Chi-squared (χ^2) distribution with $df = m - p - q$ degrees of freedom (when applied to the residuals of an ARIMA (p, d, q) model). n is numerous observations, k is many parameters tested, and ρ_k is residual autocorrelation at lag k.

Heteroscedasticity Test

Heteroscedasticity refers to a condition in which the residuals exhibit non-constant variance across observations. And the statistics test (Engle, 1982):

$$ARCH\ LM = nR^2 \quad (4)$$

Where n is numerous observations and R^2 is coefficient of determination.

Spearman's Rho Correlation

Correlation in statistics is a technique used to measure the strength and direction of the relationship between two variables (Mundir, 2012). One commonly applied method is Spearman's Rho correlation, which is particularly suitable for ordinal data or when the assumption of normality is not satisfied. The Spearman's Rho correlation coefficient is defined as follows (Myers, 2003):

$$\rho = 1 - \frac{6\sum d^2}{N(N^2-1)} \quad (5)$$

Where d is difference between rank x_1 and x_2, and N is many samples.

Copula

Copula theory was first introduced by Sklar in 1959. According to Sklar's theorem (1959), if H is the joint distribution function of random variables x and y with marginal distribution functions F and G, defined on the non-decreasing domain R,

$$F(-\infty) = G(-\infty) = 0 \text{ dan } F(\infty) = G(\infty) = 1$$

then the joint distribution is

$$H(x,y) = C\big(F(x), G(y)\big) = C(u,v) \quad (6)$$

$F(x), G(y)$ is are monotonically increasing functions with $H(x,-\infty) = H(-\infty,y) = 0$ and $H(\infty,\infty) = 1$, and C is Copula in the interval [0,1].

The joint distribution is simplified by assuming that both marginal distributions F and G are continuous, which results in C being formulated as follows (Nelsen, 2006):

$$C(u,v) = \int_0^u \int_0^v c(u,v)du, dv \quad (7)$$

Where $C(u,v)$ is the cumulative distribution function of the copula and $c(u,v)$ is the copula density function.

However, before discussing the Archimedean copula, it is necessary to first review Spearman's Rho correlation, which is commonly used as a measure of dependence in copula models. For continuous random variables X and Y associated with an Archimedean copula, the value of Spearman's Rho correlation can be calculated using the following equation (Bob, 2013):

$$\rho_{X,Y} = \rho_C = 12 \iint C(u,v)\, du\, dv - 3 \quad (8)$$

Archimedean Copula

The Archimedean copula consists of the Clayton copula, the Gumbel copula, and the Frank copula (Nelsen, 2006). The Archimedean copula function can be written as follows (Cherubini & Luciano, 2004)

$$C(u,v) = \varphi^{-1}\{\varphi(u) + \varphi(v)\} \quad (9)$$

Where $\varphi(u)$ is generator function of Archimedean Copula u, $\varphi(v)$ is generator function of Archimedean copula v, and φ^{-1} invers of φ, with φ^{-1}: [0,1].

Frank Copula

The Archimedean copula employed in this study is the Frank copula. This copula exhibits symmetric properties, meaning that the dependence between variables is treated equally in both the upper and lower tails of the distribution. The Frank copula belongs to the Archimedean family and is constructed from the following generator function (Cherubini & Luciano, 2004)

$$\varphi(t) = -ln\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right) \quad (10)$$

Then, the generator function produces the cumulative distribution function (CDF) of the Frank copula as follows (Cherubini & Luciano, 2004):

$$C(u,v) = -\frac{1}{\theta} ln\left(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right) \quad (11)$$

Where u and v is cumulative marginal distribution function of two random variables U and V in the interval [0,1], and θ is copula dependency parameter.

In the Frank copula, ρ is related to the parameter θ through the formula (Genest, 1987):

$$\rho = 1 + \frac{12}{\theta}[D_1(\theta) - D_2(\theta)] \tag{12}$$

Where $D_1(\theta)$ denotes the first-order Debye function, defined as

$$D_1(\theta) = \frac{1}{\theta}\int_0^{\theta} \frac{x}{e^x - 1} dx \tag{13}$$

$D_2(\theta)$ is denotes the second-order Debye function, defined as

$$D_2(\theta) = \frac{2}{\theta^2}\int_0^{\theta} \frac{x^2}{e^x - 1} dx \tag{14}$$

Monte Carlo Simulation

Monte Carlo simulation is a probability-based method used to approximate the outcomes of complex systems by generating and analyzing a large number of random scenarios (Glasserman, 2003). This study employs RStudio as the programming environment for data processing and statistical analysis. The dependency structure between asset returns is modeled using the Frank copula, which belongs to the Archimedean copula family and is capable of capturing non-linear and symmetric dependence. Parameter estimation and data simulation are conducted using the copula package in R. Furthermore, Monte Carlo simulation is applied to generate joint return scenarios based on the estimated Frank copula parameters, which are subsequently used to estimate the Value at Risk (VaR) at various confidence levels.

Value at Risk (VaR)

Value at Risk (VaR) was first introduced by J.P. Morgan in 1994. VaR is defined as a statistical measure that estimates the potential loss of an investment or portfolio over a specified time horizon under normal market conditions, at a given confidence level. The general formulation of VaR can be expressed as follows:

$$VaR_{\alpha} = -Quantile_{(1-\alpha)}(R_p^{(s)}) \times V \tag{15}$$

Where $R_p^{(s)}$ he portfolio return resulting from the sth Monte Carlo simulation, obtained from the combined returns of the assets in the portfolio based on the weights of each asset. And Total portfolio value is Total portfolio value.

According to Maruddani and Purbowati (2009), the VaR at a confidence level of $(1-\alpha)$ over a time horizon t, for both individual asset returns and portfolio returns, can be calculated as follows (Maruddani & Purbowati, 2012):

$$VaR_{(1-\alpha)}(t) = W_0 R^* \sqrt{t} \tag{16}$$

Where W_0 is initial investment funds (either a single asset or portfolio), R^* is α-quantile of the return distribution, and t is time period.

Data and Data Sources

The data used in this study are secondary data obtained from Yahoo Finance. Specifically, the dataset consists of stock price data of PT Telkom Indonesia Tbk (TLKM) and PT Indosat Ooredoo Hutchison Tbk (ISAT) for the period from January 15, 2024, to January 13, 2025. The data were retrieved from the Yahoo Finance website (https://finance.yahoo.com).

Data Analysis Techniques

The data analysis steps to be performed are as follows.

1. Calculate the stock return values from both stock price data sets.
2. Descriptively analyze both stock returns.
3. Check for autocorrelation effects using the Ljung-Box test.
4. Perform a heteroscedasticity test to determine whether the data has highly varied variance using the ARCH-LM test.
5. Find the Spearman's Rho correlation coefficient value.
6. Calculate the Frank Copula parameter estimate, using the Spearman's Rho correlation coefficient value obtained in step (5).
7. Calculate VaR using the Monte Carlo Simulation method with the following steps:

a. Based on the parameter estimates obtained in step (6), the Frank Copula function is used to generate random single-asset returns to simulate the return values m times.
b. Assume the portfolio weights for both assets.
c. Calculate VaR at the confidence level in the t time period.
d. Repeat step (c) m times to obtain a range of possible VaR values
8. Interpret the results obtained.

RESULTS AND DISCUSSION

Stock Price Data

This study uses stock price data from PT. Telkom Indonesia Tbk and PT. Indosat Ooredoo Hutchison Tbk for the period from January 15, 2024, to January 13, 2025, comprising 236 data points. The comparison of closing stock prices can be seen in the following Figure 1:

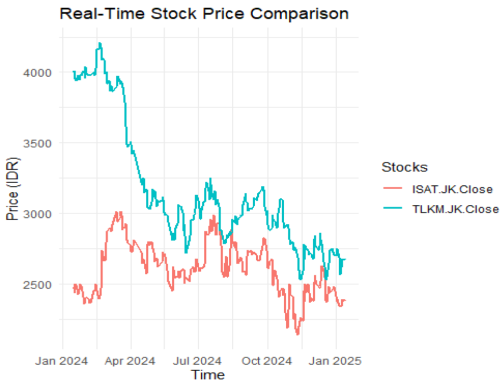

Figure 1. Closing Price Chart of TLKM and ISAT Stocks

The image shows that TLKM has a higher share price than ISAT throughout the period. TLKM experienced a significant decline in April 2024, while ISAT showed a considerable increase in March 2024.

Return On Stock Price

The overall plot of TLKM and ISAT stock returns is presented in Figure 2 and Figure 3.

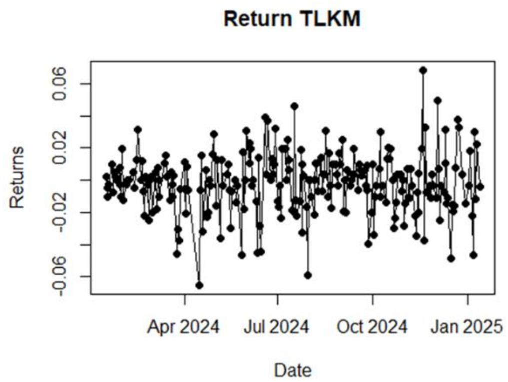

Figure 2. TLKM Return Plot

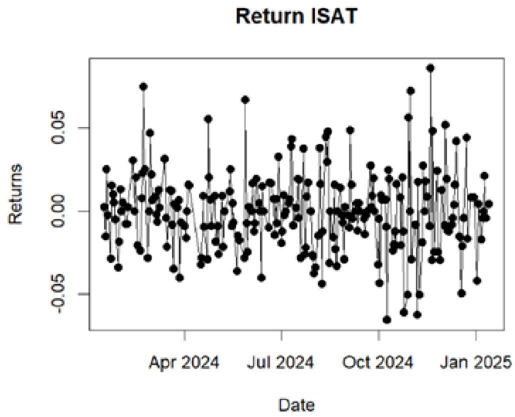

Figure 3. ISAT Return Plot

The Figure 2 and Figure 3 show that stock returns fluctuate around zero, indicating days with gains (positive returns) and losses (negative returns). The range of returns appears to vary, with some sharp spikes upward (high gains) and downward (high losses).

Descriptive Data Analysis

The descriptive statistical values of Telkom and Indosat stock returns can be seen based on the calculations in the stock return data appendix in Table 1:

Table 1. Descriptive Statistics of TLKM and ISAT Returns

	TLKM	**ISAT**
Min	-0.0652	-0.0653
Max	0.0683	0.08571
Mean	-0.0019	-0.00043
Standard Deviation	0.0191	0.02414

From the table, the minimum returns for TLKM and ISAT shares were –0.0652 (–6.52%) on April 16, 2024, and –0.0653 (–6.53%) on October 9, 2024, respectively, indicating that both stocks experienced losses on those dates. Conversely, the maximum returns for TLKM and ISAT shares were 0.0683 (6.83%) and 0.0857 (8.57%) on November 19, 2024, showing that both stocks recorded significant gains on the same day. The mean return of both stocks suggests an overall decline during the observation period. Furthermore, the standard deviation indicates the level of volatility or risk, with ISAT exhibiting a higher standard deviation than TLKM, implying that ISAT's returns were more volatile and unstable compared to TLKM.

To test whether the stock returns follow a normal distribution, the Lilliefors Corrected Kolmogorov Smirnov Normality Test is employed. This test compares the empirical distribution function of the sample with the cumulative distribution function of a specified theoretical distribution, in this case, the normal distribution. The results of the Lilliefors Corrected Kolmogorov Smirnov Normality Test for TLKM and ISAT stock returns are presented in Table 2.

Table 2. Lilliefors Corrected Kolmogorov Smirnov Test

Stock	D	$p-value$	α	**Decision**
TLKM	0.0658	0.0151	0.05	Reject H_0
ISAT	0.0673	0.0116	0.05	Reject H_0

The results in the table were obtained from Rstudio software and show that both indicate that p-values are less than the significance level (α), implying that the stock returns of TLKM and ISAT do not follow a normal distribution. Since the returns of both stocks are not normally distributed, the Frank copula is employed to model the dependence structure between the two variables, as it does not require the assumption of normality.

Autocorrelation Test

The presence of autocorrelation in stock returns can be examined using the Ljung–Box test, as presented in Table 3.

Table 3. Ljung-Box Test TLKM and ISAT Stock

Stock	Ljung-Box	
	Lag	*p-value*
TLKM	1	0,7888
	5	0,4067
	10	0,6871
	15	0,6003
	20	0,621
ISAT	1	0,7978
	5	0,2201
	10	0,4699
	15	0,6418
	20	0,3678

From the table, it can be seen that each p-value is greater than $\alpha = 0.05$. This indicates that H_0 is rejected, meaning that the returns of the two stocks are not autocorrelated and therefore the stock returns are not influenced by previous returns.

Heteroskedasticity Test

The presence of heteroscedasticity in the TLKM and ISAT stock return data can be assessed using the ARCH–LM test, the results of which are presented in the following table.

Table 4. ARCH LM Test

Stock	ARCH LM	
	p-value	DF
TLKM	0,098	10
ISAT	0,113	10

Based on the table above, it can be seen that for every p-value greater than $\alpha = 0.05$, this indicates a failure to reject H_0, which means that the returns of TLKM and ISAT stocks do not have a heteroscedasticity effect, so the returns of TLKM and ISAT have relatively constant variance, meaning there is no systematic pattern of volatility change.

Spearman’s Rho Correlation

Before estimating the parameters of the Frank Copula, first find the Spearman's Rho (ρ) correlation value. The results of the Spearman's Rho correlation can be seen in the following Table 5.

Table 5. Spearman's Rho Correlation Result

	Correlation
Spearman’s Rho	0.1364

From Table 5 above, obtained from the Rstudio software, we obtain a Spearman's Rho correlation value of 0.1364, which indicates that TLKM and ISAT stocks are positively correlated, meaning that both stocks tend to move in the same direction. A positive correlation value indicates that if the return on TLKM shares rises, the return on ISAT shares also tends to rise.

Estimation of Frank Copula Parameters

The estimated values of the Frank Copula parameters can be seen in Table 6.

Table 6. Estimation of Frank Copula

	θ
Copula Frank	0,825

From the Table 6 obtained from the Rstudio software, we obtain the Frank copula model.

$$C(u,v) = -\frac{1}{0,825} ln\left(1 + \frac{(e^{-0,825u} - 1)(e^{-0,825v} - 1)}{e^{-0,825} - 1}\right)$$

A value of θ > 0 indicates that the copula exhibits positive dependence, implying that when the price of TLKM shares increases, the price of ISAT shares also tends to increase. The function $C(u,v)$ represents the joint probability distribution of two random variables after being transformed into a uniform distribution. Meanwhile, u and v denote the values of these random variables, each transformed into the interval [0,1].

To examine whether the empirical data is consistent with the Frank Copula, a scatterplot analysis was conducted. At this stage, random numbers were generated from the Frank Copula using the estimated parameter obtained earlier. The simulation was carried out with m = 10.000 replications to ensure stability of the results. The scatterplot of the simulated data is presented in the following figure.

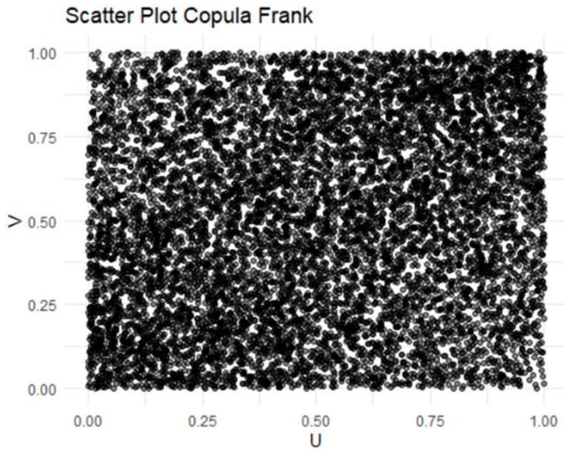

Figure 4. Scatterplot of Simulation Data

The figure shows that the points are not only concentrated at the bottom end but also at the top end. This indicates the presence of upper and lower tail dependence, namely Frank copula. The scatterplot reflects the symmetrical nature of Frank copula. This shows that the relationship between variables u and v is balanced and does not depend on the order of the variables.

Value at Risk Estimation

The confidence levels used in this study are 90%, 95%, and 99%. The VaR values of TLKM and ISAT stock returns can be seen in the following Table 7 .

Table 7. Value at Risk (VaR)

	Confidence Levels		
	90%	**95%**	**99%**
VaR	-0.0222	-0.0281	-0.0383

Table 7 informs that for 90% confidence level, the VaR value of −0.0222 indicates that there is a 90% probability that the one-day loss will not exceed 2.22% of the investment value, while there remains a 10% probability that losses may be equal to or greater than this threshold. At the 95% confidence level, the VaR value of −0.0281 implies that the potential loss will not exceed 2.81% with a probability of 95%, leaving a 5% chance of larger losses. Meanwhile, at the 99% confidence level, the VaR value of −0.0383 suggests that extreme losses exceeding 3.83% may occur with a probability of only 1%. To illustrate the economic implication of these estimates, an investment of Rp 100,000,000 may experience a potential one-day loss of up to Rp 2,220,000 at the 90% confidence level, Rp 2,810,000 at the 95% confidence level, and Rp 3,830,000 at the 99% confidence level.

However, VaR estimation alone does not guarantee the accuracy of risk measurement. Therefore, this study conducts backtesting using Kupiec’s Proportion of Failures (POF) test to evaluate whether the observed frequency of VaR violations is consistent with the expected failure rates implied by the selected confidence levels.

Kupiec's Proportion of Failures (POF) Test Results

Table 8. Kupiec's POF Backtesting Results

Convidence Level	LR_{POF}	p_{value}	Decision
90%	0.1044	0.7466	Fail to Reject H_0 (VaR Valid)
95%	4.0003	0.0455	Reject H_0 (VaR Not Valid)
99%	8.4388	0.0037	Reject H_0 (VaR Not Valid)

The results of Kupiec's Proportion of Failures (POF) test presented in Table 8 indicate varying levels of VaR model adequacy across different confidence levels. At the 90% confidence level, the LR $_{POF}$ statistic is 0.1044 with a p-value of 0.7466, which is greater than the 5% significance level. Therefore, the null hypothesis cannot be rejected, indicating that the observed number of VaR violations is consistent with the expected failure rate. This suggests that the VaR model provides reliable risk estimates at the 90% confidence level.

In contrast, at the 95% confidence level, the LR_{POF} value increases to 4.0003 with a p-value of 0.0455, leading to the rejection of the null hypothesis at the 5% significance level. This result implies that the frequency of observed VaR violations exceeds the theoretical expectation, indicating that the VaR model tends to underestimate risk at this confidence level. A similar conclusion is observed at the 99% confidence level, where the LR_{POF} statistic reaches 8.4388 with a p-value of 0.0037. The rejection of the null hypothesis at this level suggests a significant underestimation of extreme losses by the VaR model. Overall, these findings indicate that while the VaR model performs adequately under moderate risk conditions (90% confidence level), its accuracy deteriorates at higher confidence levels, particularly in capturing tail risk. This highlights the importance of backtesting in VaR analysis and suggests that alternative risk modeling approaches may be required to better account for extreme market movements.

CONCLUSION

Based on the results and discussion of the Value at Risk (VaR) calculation using the Frank Copula with Spearman's Rho correlation for the shares of PT Telkom Indonesia Tbk and PT Indosat Ooredoo Hutchison Tbk during the period January 15, 2024 – January 13, 2025, the following conclusions can be drawn:

1. The Frank Copula parameters were estimated using the Spearman's Rho correlation coefficient, resulting in $\hat{\theta} = 0.825$and $\rho = 0.136$. Thus, the Frank Copula model obtained is:
$$C(u,v) = -\frac{1}{0,825} ln\left(1 + \frac{(e^{-0,825u} - 1)(e^{-0,825v} - 1)}{e^{-0,825} - 1}\right)$$
2. Using the Frank Copula, the one-day VaR estimates at the 90%, 95%, and 99% confidence levels are -0.0222, -0.0281, and -0.0383, respectively. Backtesting results using Kupiec's POF test indicate that the VaR model is statistically valid only at the 90% confidence level, while it fails to accurately capture risk at higher confidence levels.

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