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COMPARATIVE ANALYSIS OF UNFUNDED ACTUARIAL LIABILITY BASED ON HULL–WHITE INTEREST RATE ESTIMATION USING ORDINARY LEAST SQUARES AND JACKKNIFE

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ABSTRACT

Pension programs are designed to provide financial security after retirement, requiring accurate actuarial valuation to ensure funding adequacy. A key determinant of actuarial liabilities is the interest rate assumption, which directly affects the present value of future pension obligations and the level of unfunded actuarial liability (UAL). Despite its importance, most pension valuation studies rely on deterministic interest rates, while empirical evidence on the use of stochastic interest rate models combined with robust parameter estimation techniques remains limited. This study addresses this gap by evaluating actuarial liability adequacy using the Frozen Initial Liability (FIL) method under a stochastic interest rate framework. The Hull–White one-factor model is employed to capture the dynamic behavior of interest rates, with parameters estimated using Ordinary Least Squares (OLS) and the Jackknife method. The Jackknife approach is introduced to improve estimation robustness, particularly in the presence of small samples and influential observations. Empirical results show that the Jackknife method produces an average interest rate of 0.0678 with a Mean Absolute Percentage Error (MAPE) of 24.4%, while OLS yields an average rate of 0.0665 with a MAPE of 26.1%. Both approaches result in negative UAL values, indicating a fully funded pension scheme with a surplus position. However, the surplus obtained under the Jackknife estimation is lower despite the higher interest rate estimate, suggesting an inverse relationship between interest rates and surplus levels within the FIL framework.

Keywords: *Frozen Initial Liability, Hull-White, Jackknife, Ordinary Least Square, Unfunded Actuarial Liability.*

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INTRODUCTION

Retirement is a phase in which individuals cease working upon reaching the statutory retirement age or by personal decision. At this stage, individuals are entitled to pension benefits or severance payments to ensure financial security in old age (Lesmana, 2015). Pension fund programs represent long-term financial commitments by employers in recognizing employees' contributions throughout their working lives. In Indonesia, pension fund management is conducted by both public and private institutions, including PT Jamsostek, PT Asabri, and PT TASPEN, under the supervision of BPJS Ketenagakerjaan (Ahmad, Alhusain, & Silalahi, 2015). As the number of retirees continues to increase, pension fund managers face growing challenges in ensuring that sufficient funds are available to meet long-term benefit obligations.

A pension fund functions as a long-term savings mechanism that provides income security after retirement. Pension benefits are accumulated during the employment period and remain payable even in cases of disability or death prior to retirement. While pension schemes enhance job security and employee motivation, inadequate funding may lead to deficits that threaten the sustainability of benefit payments (Marwa, 2020). Consequently, pension obligations must be recognized as actuarial liabilities that should be fully funded at retirement. Any shortfall between available assets and actuarial liabilities, referred to as Unfunded Actuarial Liability (UAL), poses a significant financial risk and highlights the importance of accurate actuarial valuation and funding strategies. This study focuses on the problem of measuring and managing UAL within defined-benefit pension schemes under uncertain interest rate conditions.

Various actuarial funding methods have been developed to address pension valuation, particularly for group-based pension programs. One widely used approach is the Frozen Initial Liability (FIL) method, which is derived from the Entry Age Normal (EAN) method and assumes constant normal contributions for all participants regardless of retirement age. Previous studies have applied the FIL method primarily under deterministic interest rate assumptions, which may fail to capture the inherent volatility of financial markets. To address this limitation, stochastic interest rate models have been introduced, among which the Hull-White model has gained prominence due to its ability to reflect mean reversion and align with the observed term structure of interest rates (Alfikri, Satyahadewi, & Perdana, 2020).

In terms of the state of the art, existing research on stochastic interest rate modeling has largely concentrated on parameter estimation and model behavior, often employing Ordinary Least Squares (OLS) as the primary estimation technique. Some studies have introduced resampling-based methods such as the Jackknife to improve robustness and reduce sensitivity to outliers (Ariani, Nasution, & Yuniarti, 2017). Empirical evidence suggests that the Jackknife method can produce lower volatility estimates in the Cox–Ingersoll–Ross (CIR) model (Yunizar, 2019), while comparable parameter behavior has been observed for the Hull-White model (Choliqa, 2022). However, these studies predominantly evaluate estimation performance in isolation and do not extend the analysis to actuarial funding outcomes.

What remains insufficiently explored is how different parameter estimation methods within stochastic interest rate models influence actuarial liability calculations and, in particular, the magnitude and dynamics of Unfunded Actuarial Liability under specific pension funding methods such as FIL. The direct linkage between interest rate estimation accuracy and pension funding adequacy has not been explicitly addressed in prior studies, leaving pension fund managers with limited empirical guidance when selecting estimation techniques for actuarial valuation (Pangestu & Mahrani, 2023).

Accordingly, the objective of this study is to conduct a comparative analysis of Unfunded Actuarial Liability using the Frozen Initial Liability (FIL) method under a Hull-White stochastic interest rate framework, with parameters estimated using both Ordinary Least Squares and Jackknife methods. By integrating robust interest rate estimation techniques into actuarial funding analysis, this study aims to evaluate how differences in estimation approaches affect actuarial surplus or deficit outcomes and to provide practical insights for more accurate and sustainable pension fund management.

Despite the extensive use of stochastic interest rate models and actuarial funding methods in pension valuation, existing studies tend to address these components separately. Prior research has focused either on interest rate modeling and parameter estimation performance, or on actuarial funding methods under fixed or simplified interest rate assumptions. However, there is limited empirical evidence on how different parameter estimation techniques within a stochastic interest rate framework translate into actuarial funding outcomes, particularly Unfunded Actuarial Liability (UAL) under the Frozen Initial Liability (FIL) method. Accordingly, this study contributes by integrating the Hull–White stochastic interest rate model with the FIL actuarial funding framework and by comparing OLS and Jackknife parameter estimation methods in terms of their implications for actuarial liabilities and funding adequacy. Rather than proposing a new estimation method, this study provides empirical insight into how estimation choices affect pension funding results in practice.

MATERIALS AND METHODS

Data

This study utilizes two primary datasets. The first dataset comprises the Bank Indonesia policy interest rate (BI Rate) observed over the period 2000–2023, resulting in a total of 24 annual observations that capture long-term interest rate dynamics in Indonesia. The second dataset relates to Civil Servants, consisting of a sample of 50 individuals classified under grade 3A. For this group, the variables analyzed include entry age into civil service, statutory retirement age, and base salary, which collectively serve as key inputs for modeling employment duration and income-related financial outcomes.

Mortality Table and Commutation Symbol

A mortality table summarizes the pattern of deaths within a population over a specified period and provides the probabilities associated with survival and death at different ages. In this table, l_x denotes the number of individuals who remain alive at exact age x, while d_x represents the number of individuals who die at that age (Pangestu & Mahrani, 2023). Mortality tables serve as a fundamental tool in actuarial science, forming the basis for the valuation of life-contingent financial products. The probability that an individual aged x will survive for an additional t years, denoted by ${}_tp_x$, is expressed as:

$$ {}_t\boldsymbol{p}_{\boldsymbol{x}} = \frac{\boldsymbol{l}_{\boldsymbol{x}+\boldsymbol{t}}}{\boldsymbol{l}_{\boldsymbol{x}}} \tag{1} $$

This probability reflects the proportion of individuals aged $x$who are expected to be alive at age $x + t$. Conversely, the probability that an individual aged x will die before reaching age $x + t$, denoted by ${}_tq_x$, is given by:

$$ {}_t\boldsymbol{q}_{\boldsymbol{x}} = \frac{\boldsymbol{l}_{\boldsymbol{x}} - \boldsymbol{l}_{\boldsymbol{x}+\boldsymbol{t}}}{\boldsymbol{l}_{\boldsymbol{x}}} \tag{2} $$

This measure represents the complement of the survival probability over the same time interval and is essential in modeling mortality risk.

In actuarial calculations, commutation functions are commonly employed to simplify the evaluation of future cash flows associated with insurance benefits and annuity payments. These functions introduce specialized notations that facilitate efficient computation, particularly for payments made at the beginning of each period, denoted by $\ddot{a}$ (Andriananda & Maulana, 2023). Among the commutation symbols derived from mortality tables is:

$$ \boldsymbol{D}_{\boldsymbol{x}} = \boldsymbol{v}^{\boldsymbol{x}} \boldsymbol{l}_{\boldsymbol{x}} \tag{3} $$

where $\boldsymbol{v}^{\boldsymbol{x}}$ represents the discount factor accounting for the time value of money. The symbol $\boldsymbol{D}_{\boldsymbol{x}}$ therefore combines both survival probabilities and interest discounting. Furthermore, the commutation symbol $\boldsymbol{N}_{\boldsymbol{x}}$ is defined as the cumulative sum of $\boldsymbol{D}_{\boldsymbol{x}}$ over future ages and is expressed as

$$ \boldsymbol{N}_{\boldsymbol{x}} = \sum_{\boldsymbol{t}=\boldsymbol{0}}^{\boldsymbol{\omega}} \boldsymbol{D}_{\boldsymbol{x}+\boldsymbol{t}} \tag{4} $$

The parameter ω denotes the limiting age, which corresponds to the maximum attainable age assumed in the construction of the mortality table. These commutation symbols play a critical role in actuarial valuation by enabling concise and systematic calculations of life insurance premiums, reserves, and pension benefits.

Whole Life Annuity Due

An annuity is a payment of a predetermined amount made at regular intervals or over a specified period. A whole life annuity, which is received by pension program participants from the beginning of the period until the participant's death and is paid at the start of the year (Caraka, 2016), can be formulated as follows

$$ \ddot{\boldsymbol{a}}_{\boldsymbol{x}} = \sum_{\boldsymbol{t}=\boldsymbol{0}}^{\infty} \boldsymbol{v}^{\boldsymbol{t}}\, {}_t\boldsymbol{p}_{\boldsymbol{x}} = \frac{\boldsymbol{N}_{\boldsymbol{x}}}{\boldsymbol{D}_{\boldsymbol{x}}} \tag{5} $$

The payment of a whole life annuity is not limited to annual periods but can also be made monthly or other periodic. If a payment of 1 is made at the beginning of each period with a frequency of m Times per year, the equation can be formulated as follows.

$$\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m} \quad (6)$$

Salary and Benefit Function

Salary notation is important for estimating future salaries since pension benefits depend on the salary growth of participants. The annual salary of a participant aged x is denoted as s_x, while S_x represents the accumulated salary from the entry age e to x (Utami, Wilandari, & Wuryandari, 2012) which can be formulated as follows

$$s_{x+t} = s_x(1+s)^t \quad (7)$$

Thus, a participant's total income or salary before retirement is formulated as follows.

$$S_{r-1} = \sum_{x=e}^{r-1} s_x \quad (8)$$

The benefit function determines the benefits paid upon retirement, termination of employment, disability, and death. The amount of retirement benefits at the age r is based on the participant's average salary during employment from entry-age e to retirement age r, formulated as follows:

$$B_r = kS_{r-1} \quad (9)$$

Brownian Motion and Wiener Process

A stochastic process $\{X(t), t \in T\}$ is a collection of random variables, meaning that for every $t \in T$, $X(t)$ is a random variable. Since the index t is often interpreted as time, $X(t)$ represents the state of the process at time t. For example, $X(t)$ may denote the number of customers entering a supermarket at time $t$or the total sales recorded in a market at time t. The set T is referred to as the index set of the process. When T is a countable set, the stochastic process is called a discrete-time stochastic process. For instance, $\{X_n, n = 0,1, \ldots\}$ is a discrete-time stochastic process indexed by non-negative integers, such as months. Conversely, when T is an interval on the real line, the stochastic process is called a continuous-time stochastic process. As an example, $\{X(t), t \geq 0\}$ represents a continuous-time stochastic process indexed by non-negative real numbers (Lestari & Mahrani, 2024).

Brownian motion refers to the random and continuous movement of particles suspended in a fluid (liquid or gas). It was first observed in 1827 by the Scottish botanist Robert Brown, who noted that pollen particles suspended in water moved irregularly in random directions, with the intensity of motion increasing as temperature rose (Taylor & Karlin, 1998). In the early 1900s, Louis Bachelier extended Brown's observations by providing the first mathematical formulation of random motion and applying it to model stock price fluctuations in the Paris Stock Exchange. Subsequently, in the 1920s, Norbert Wiener developed a rigorous probabilistic framework for this model, which is now known as Brownian motion or the Wiener process (Wiersema, 2008). The Wiener process is a special type of continuous-time Markov stochastic process with zero mean increments and unit variance per unit time. It is widely used in physics to describe particle motion under molecular collisions and has become a fundamental building block in stochastic modeling (Hull, 2009). A stochastic process $W(t)$is called a Brownian motion if it satisfies the following properties: (i) $W(0) = 0$; (ii) it has independent increments over non-overlapping time intervals; and (iii) the increment over any interval of length u, from time $t$to $t + u$, is normally distributed with mean zero and variance equal to u.

Itô Process

An Itô process is a generalized Wiener process characterized by two parameters, $a$and b, which are functions of the underlying state variable $r(t)$and time t (Hull, 2009). This process forms the foundation of stochastic calculus and is widely applied in financial and actuarial modeling. The evolution of an Itô process is defined through the Itô integral, which captures the accumulation of stochastic effects over time. For a simple (stepwise constant) adapted process $r(t)$, the Itô integral over the interval $[0, T]$is defined as

$$\int_0^T r(t)\, dW(t) = \sum_{i=0}^{n-1} r_i \,[W(t_{i+1}) - W(t_i)], \quad (10)$$

where $\{t_i\}_{i=0}^{n}$ is a partition of the interval $[0, T]$, r_i denotes the value of $r(t)$ on the subinterval $[t_i, t_{i+1})$, and $W(t)$ represents a standard Wiener process. For more general stochastic processes, the Itô integral is defined as the limit in mean square of such sums. In its simplest form, the Itô integral satisfies several important properties. First, linearity holds: if $r(t)$ and $Y(t)$ are simple stochastic processes and a and b are constants, then

$$\int_0^T (a\,r(t) + b\,Y(t))\,dW(t) = a\int_0^T r(t)\,dW(t) + b\int_0^T Y(t)\,dW(t). \tag{11}$$

Second, the expectation of the Itô integral is zero, provided that the integrand is square-integrable:

$$\mathrm{E}\left[\int_0^T r\,(t)\,dW(t)\right] = 0. \tag{12}$$

Third, the Itô integral satisfies the isometry property, which relates its second moment to the integral of the squared integrand:

$$\mathrm{E}\left[\left(\int_0^T r\,(t)\,dW(t)\right)^2\right] = \int_0^T \mathrm{E}\,[r^2(t)]dt. \tag{13}$$

This property, known as th**e** Itô isometry, is fundamental in the analysis of stochastic differential equations and provides a direct link between stochastic integrals and classical Lebesgue integrals.

Euler-Maruyama Method

The Euler–Maruyama method is a numerical approximation approach designed to solve stochastic differential equations (SDEs) when analytical solutions are not readily available. This method extends the classical Euler scheme developed by Leonhard Euler, and its core idea is to approximate continuous-time stochastic processes through discretization of the time domain into sufficiently small intervals (Siahaan, Mahrani & Sofia, 2024). Within the framework of SDEs, the Euler–Maruyama method estimates the value of a stochastic process $X(t)$at discrete time instants t_i, such that the continuous process is approximated by $X(t_i) \approx X_i$for $i = 0,1,\dots,N$ over the interval $[0, T]$. The general update rule of the Euler–Maruyama scheme can be written as:

$$X_{i+1} = X_i + b(t_i, X_i)\Delta t + \sigma(t_i, X_i)\sqrt{\Delta t}\,Z_i \tag{14}$$

where $\Delta t = t_{i+1} - t_i$ and Z_i denotes a standard normally distributed random variable, $Z_i \sim \mathcal{N}(0,1)$. As an illustrative example, consider the following stochastic differential equation:

$$dX(t) = dW(t) \tag{15}$$

subject to the initial condition $X(0) = W(0) = 0$. The exact solution of this equation is $X(t) = W(t)$, which corresponds to a Wiener process. Using the Euler–Maruyama scheme, this process is approximated in discrete time by

$$X_{i+1} = X_i + \sqrt{\Delta t}\,Z_i \tag{16}$$

Moreover, the Euler–Maruyama method can be employed to discretize Equation (2.20) by replacing the continuous-time formulation with a discrete-time representation. This procedure leads to the following approximation:

$$r(t_{i+1}) = \theta(t)\Delta t + (1 - a\Delta t)r(t_i) + \sigma\Delta W_i \tag{17}$$

where $\Delta W_i = \sqrt{\Delta t}\,Z_i$represents the increment of the Wiener process over the i-th time step.

Hull-White Model

The Hull-White interest rate model is a no-arbitrage interest rate model that accurately reflects the current term structure of interest rates. The Hull-White interest rate model is often called the Hull-White Extended Vasicek model because it extends the Vasicek model, making it known as the Hull-White model. The Hull-White interest rate model can be formulated as follows (Hull, 2009) :

$$\boldsymbol{dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t)} \tag{18}$$

The equation above, when integrated, yields:

$$\boldsymbol{r(t) = r(0)e^{-at} + \frac{\theta(t)}{a}(1 - e^{-at}) + \sigma e^{-at}\int_0^t e^{as}dW(s)} \tag{19}$$

By applying the properties of the Itô integral, the expectation and variance can be derived as follows.

$$E[r(t)] = r(0)e^{-at} + \frac{\theta(t)}{a}(1 - e^{-at}) \tag{20}$$

And

$$var[r(t)] = \frac{\sigma^2}{2a}(1 - e^{-at}) \tag{21}$$

The most important feature of this model is mean reversion, which can be described as the tendency of the short-term interest rate $\boldsymbol{r(t)}$to fluctuate around a long-term average level, or equivalently, the tendency of interest rates to move within a bounded range. Consequently, short-term interest rates exhibit a tendency to revert toward their long-run mean. When interest rates approach zero, volatility also tends to decline, reducing the impact of random fluctuations and ensuring that interest rates remain positive. Conversely, when interest rates are high, volatility tends to increase, which is a desirable characteristic of interest rate models (Lestari & Mahrani, 2024).

Although the Hull–White model is formulated in continuous time as a stochastic differential equation, empirical estimation requires discretization because interest rate data are observed at discrete time intervals. In this study, the integrated form of the Hull–White model is approximated using a discrete-time representation consistent with annual observations. This discretized specification enables parameter estimation using Ordinary Least Squares (OLS). The Jackknife method is subsequently applied as a resampling-based extension of the same discrete estimation framework, where model parameters are repeatedly re-estimated by systematically omitting one observation at a time. Thus, both OLS and Jackknife estimations are conducted on the discrete approximation of the continuous-time Hull–White process.

Ordinary Least Square **Method**

The Ordinary Least Squares (OLS) method is one of the most widely used estimation techniques in statistical modeling and econometric analysis. OLS is employed to estimate the parameters of a linear regression model by minimizing the sum of squared differences between observed values and their corresponding fitted values. Due to its simplicity and strong theoretical properties under classical assumptions, OLS remains a fundamental tool in empirical research across economics, finance, and actuarial science (Gujarati & Porter, 2009). Consider the following linear regression model:

$$\boldsymbol{y = X\beta + e} \tag{22}$$

In this formulation, $\mathbf{y}$denotes an $\boldsymbol{n \times 1}$vector of observed values of the dependent variable, $\mathbf{X}$ represents an $\boldsymbol{n \times k}$ matrix of explanatory variables including a constant term, $\boldsymbol{\beta}$ is a $\boldsymbol{k \times 1}$ vector of unknown parameters to be estimated, and $\boldsymbol{\varepsilon}$ is an $\boldsymbol{n \times 1}$ vector of random error terms capturing unobserved influences. The equation above is minimized by differentiation, as the least squares estimator for the parameter estimation. The OLS estimator is obtained by minimizing the objective function:

$$\boldsymbol{S(\beta) = \sum_{i=1}^{n} \varepsilon_i^2 = (y - X\beta)'(y - X\beta)}. \tag{23}$$

Taking the first-order condition with respect to $\boldsymbol{\beta}$ and solving yields the closed-form OLS estimator:

$$\boldsymbol{\hat{\beta}_{ols} = (X'X)^{-1}X'y} \tag{24}$$

Under the Gauss–Markov assumptions—namely linearity, exogeneity, homoscedasticity, and the absence of autocorrelation—the OLS estimator is unbiased and efficient, and it possesses the property of being the Best Linear Unbiased Estimator (BLUE) (Gujarati & Porter, 2009). When the error terms are further assumed to be normally distributed, statistical inference such as hypothesis testing and confidence interval estimation can be conducted using the estimated variance–covariance matrix of the parameters (Wooldridge, 2016).

In financial and actuarial applications, the OLS method is frequently used for parameter estimation in interest rate models, asset pricing equations, and risk factor analysis. Despite its widespread use, OLS estimation may be sensitive to small sample sizes, heteroscedasticity, or serial correlation, which are common characteristics of financial time series data. Consequently, robustness-enhancing techniques or alternative estimators are often employed to complement OLS results (Wooldridge, 2016).

Jackknife Method

The Jackknife method is a resampling-based statistical technique used to evaluate the bias, variance, and robustness of parameter estimates, particularly when dealing with small sample sizes or estimators that are sensitive to individual observations **(Siahaan, Mahrani, & Sofia, 2024)**. The core principle of this method is to repeatedly re-estimate the parameter of interest by systematically excluding one observation at a time from the original sample. Let $\hat{\boldsymbol{\theta}}$ denote an estimator of the parameter $\boldsymbol{\theta}$ computed from a sample of size $\boldsymbol{n}$. The Jackknife procedure generates a set of leave-one-out estimators, each based on a subsample of size $\boldsymbol{n-1}$, which allows for an assessment of the stability of the original estimator. By aggregating these leave-one-out estimates, the Jackknife method provides both a bias-corrected estimator and an estimate of the sampling variance, thereby improving the reliability of inference when classical assumptions may not be fully satisfied. Owing to its conceptual simplicity and relatively low computational cost, the Jackknife method has been widely applied in econometric, financial, and actuarial studies to enhance estimator robustness and to mitigate the influence of individual data points

The following is the procedure for the Jackknife method used to estimate parameters by removing one data point and randomly sampling.

$$\boldsymbol{y^i} = \begin{bmatrix} y_1^i \\ y_2^i \\ \vdots \\ y_{n-1}^i \end{bmatrix}, \boldsymbol{X^i} = \begin{bmatrix} 1 & X_{11}^i & X_{12}^i & \cdots & X_{1j}^i \\ 1 & X_{21}^i & X_{22}^i & \cdots & X_{2j}^i \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{(n-1)1}^i & X_{(n-1)2}^i & \cdots & X_{(n-1)j}^i \end{bmatrix}, \boldsymbol{e^i} = \begin{bmatrix} e_1^i \\ e_2^i \\ \vdots \\ e_{n-1}^i \end{bmatrix} \quad (25)$$

Let y^idenote the vector of the dependent variable after excluding the i-th observation, X^irepresent the corresponding matrix of independent variables with the i-th row removed, and e^idenote the vector of residuals obtained from this reduced sample. Using the least squares approach, the parameter estimate $\hat{\beta}^i$for the i-th Jackknife sample is obtained by minimizing the sum of squared residuals, which yields the estimator given :

$$\hat{\boldsymbol{\beta}}^{\boldsymbol{i}} = (\boldsymbol{X^{i\prime}X^i})^{-1}\boldsymbol{X^{i\prime}y^i} \quad (26)$$

The symbol $\boldsymbol{i}$ represents the row in the matrix, where $\boldsymbol{i = 1, \dots, n}$ resulting in the Jackknife parameter estimates $\hat{\boldsymbol{\beta}}^{\boldsymbol{1}}, \hat{\boldsymbol{\beta}}^{\boldsymbol{2}}, \dots, \hat{\boldsymbol{\beta}}^{\boldsymbol{n}}$ Thus, the Jackknife parameter estimate can be obtained from the average value of each parameter. $\hat{\boldsymbol{\beta}}^{\boldsymbol{1}}, \hat{\boldsymbol{\beta}}^{\boldsymbol{2}}, \dots, \hat{\boldsymbol{\beta}}^{\boldsymbol{n}}$ As follows [8]:

$$\hat{\boldsymbol{\beta}} = \sum_{i=1}^{n} \frac{\hat{\boldsymbol{\beta}}^{\boldsymbol{i}}}{\boldsymbol{n}} \quad (27)$$

This averaging process serves to reduce estimation bias and produces a more robust parameter estimate compared to that obtained from a single full-sample estimation.

Mean Absolute Percentage Error (MAPE)

The Mean Absolute Percentage Error (MAPE) is a widely used accuracy measure for evaluating the performance of forecasting and predictive models. MAPE quantifies the average magnitude of prediction errors in percentage terms, thereby providing an intuitive interpretation of model accuracy relative to the actual observed values (Maricar, 2019). Given a set of actual observations $\boldsymbol{y_t}$and corresponding predicted values $\hat{\boldsymbol{y}}_{\boldsymbol{t}}$ for $\boldsymbol{t = 1, 2, \dots, n}$, the MAPE is defined as:

$$\boldsymbol{MAPE} = \frac{\boldsymbol{100\%}}{\boldsymbol{n}} \sum_{i=1}^{n} \left| \frac{\boldsymbol{Y_i - \hat{Y}_i}}{\boldsymbol{Y_i}} \right| \quad (28)$$

MAPE measures the average absolute deviation between predicted and actual values as a percentage of the actual values. Lower MAPE values indicate better predictive accuracy, with values closer to zero implying a higher degree of model fit.

In empirical applications, MAPE is particularly appealing due to its scale-independent nature, allowing for straightforward comparison across different models or datasets. However, it is important to note that MAPE may be undefined or unstable when actual values $y_t$are close to zero, and it may disproportionately penalize negative errors. Despite these limitations, MAPE remains a popular metric in financial, actuarial, and time-series forecasting studies due to its simplicity and interpretability.

Frozen Initial Liability Method

The Frozen Initial Liability (FIL) method is a pension funding method commonly used to calculate pension funding for a specific group (Alfikri, Satyahadewi, & Perdana, 2020). The Frozen

Initial Liability method adapts the Entry Age Normal method, which begins by determining the normal contribution amount for all participants. The assumption used in this method is that the normal contribution for each participant remains the same for all members within the group. (Pangestu & Mahrani, 2023) The normal contribution formula for the FIL method is expressed as follows:

$$NC_t^j = \frac{1}{n_t}\sum_{j\in A_t} B_r^j \ddot{a}_r^{(12)} \frac{D_r}{N_e - N_r} \tag{29}$$

The actuarial liability is the pension funds that should be accumulated for future pension payments. The actuarial liability under the Frozen Initial Liability (FIL) method can be formulated as follows (Gajek & Ostaszewski, 2004):

$$\begin{aligned} AL_{t+1} = {} & AL_t(1+i) - \left(\sum_{j\in T}\widetilde{PVFB}_{t+1}^j - \sum_{j\in A_t} q_x \widetilde{PVFB}_{t+1}^j\right) - \sum_{j\in R}\widetilde{PVFB}_{t+1}^j \\ & + \sum_{j\in A_{t+1}} \Delta B_t^j \ddot{a}_y^{(12)} \frac{D_r}{D_{e+1}} - \frac{1}{n_{t+1}} NC_{t+1} \sum_{j\in A_{t+1}} \frac{N_{e+1} - N_r}{D_{e+1}} + (1 \\ & + i)\frac{1}{n_t} NC_t \sum_{j\in A_t} \frac{N_e - N_r}{D_e} \end{aligned} \tag{30}$$

Unfunded Actuarial Liability

Unfunded Actuarial Liability (UAL) refers to the portion of an actuarial liability that is not covered by the actuarial value of plan assets. It represents the funding shortfall of a pension or long-term benefit plan when the present value of promised future benefits exceeds the assets accumulated to finance those obligations. UAL is a key indicator of the financial sustainability and solvency of defined benefit pension schemes and long-term care programs (Gajek & Ostaszewski, 2004). Thus, UAL is formulated as follows:

$$UAL_{t+1} = (UAL_t + TNC_t)(1+i) - C - I_c \tag{31}$$

where TNC_t denotes the total normal cost at time t, i represents the actuarial interest rate, C denotes the contribution made during the period, and I_c represents interest credited on contributions. In this study, the contribution term C is assumed to be zero, reflecting a scenario in which no additional funding is made during the valuation period. This assumption allows the analysis to focus on the intrinsic dynamics of the unfunded actuarial liability driven by benefit accruals and interest accumulation, thereby highlighting the potential growth of UAL in the absence of corrective funding measures.

RESULTS AND DISCUSSION

Hull-White Interest Rate Modelling

The Hull–White interest rate modeling in this study is initiated by estimating the regression parameter $\boldsymbol{\beta}$derived from Equation (19), which results from the discretization of the continuous-time Hull–White model. Parameter estimation is conducted using two approaches, namely the Ordinary Least Squares (OLS) method and the Jackknife method, in order to assess both estimation efficiency and robustness. The estimated values of the regression parameters $\hat{\boldsymbol{\beta}}_0$ and $\hat{\boldsymbol{\beta}}_1$ obtained from these methods are reported in Table 1. The results indicate that the OLS and Jackknife estimates are very close, suggesting that the regression relationship underlying the discretized model is stable and not overly influenced by individual observations.

Table 1. Value of Parameter $\boldsymbol{\beta}$

Parameter	OLS	*Jackknife*
$\hat{\beta}_0$	0.0224	0.0225
$\hat{\beta}_1$	0.6671	0.6664

The estimated values of $\boldsymbol{\beta}$ are subsequently used to derive the structural parameters of the Hull–White model, namely the speed of mean reversion $\boldsymbol{a}$, the long-term mean level $\boldsymbol{\theta}$, and the volatility parameter $\boldsymbol{\sigma}$. The resulting parameter estimates obtained from both estimation methods are presented in Table 2. The mean reversion parameter $\hat{\boldsymbol{a}}$ reflects the rate at which the short-term interest rate adjusts toward its long-term equilibrium, while $\hat{\boldsymbol{\theta}}$ represents the long-term mean level to which the interest rate converges over time. Meanwhile, the volatility parameter $\hat{\boldsymbol{\sigma}}$ captures the magnitude of random fluctuations in the short-term interest rate process.

Table 2. Parameter Estimation

Parameter	*OLS*	*Jackknife*
$\hat{a}$	0,3328	0,3335
$\hat{\theta}$	0,0224	0,0225
$\hat{\sigma}$	0,0731	0,0731

As shown in Table 2, the parameter estimates produced by the OLS and Jackknife methods are largely consistent. In particular, the estimated long-term mean $\hat{\boldsymbol{\theta}}$ and volatility $\hat{\boldsymbol{\sigma}}$ are identical under both methods, indicating strong robustness with respect to the estimation technique. The Jackknife-based estimate of the mean reversion speed $\hat{\boldsymbol{a}}$is slightly higher than that obtained from OLS, reflecting a minor adjustment after accounting for the influence of individual observations.

It should be noted that the interest rate dataset employed in this study consists of 24 annual observations of the Bank Indonesia policy rate, which represents a relatively small sample size. This limitation may affect statistical power and the robustness of parameter estimation. Furthermore, the OLS approach relies on classical assumptions such as linearity, independence, and homoscedasticity of error terms, which may not fully hold for interest rate data that are often characterized by persistence and volatility clustering. For this reason, the Jackknife method is applied not as a substitute for OLS, but as a robustness-enhancing technique that mitigates sensitivity to individual observations, particularly in small-sample settings. Consequently, the estimated Hull–White parameters should be interpreted as empirical results conditional on the available data and the modeling assumptions adopted in this study.

The error measurement of the Hull–White model is conducted to assess the accuracy of the model in replicating observed interest rate movements. Model performance is evaluated using the Mean Absolute Percentage Error (MAPE), which measures the average magnitude of deviations between the observed Bank Indonesia (BI) Rate and the interest rates generated by the Hull–White model. The MAPE is calculated separately for each parameter estimation method, namely Ordinary Least Squares (OLS) and Jackknife, using the corresponding simulated interest rate paths.

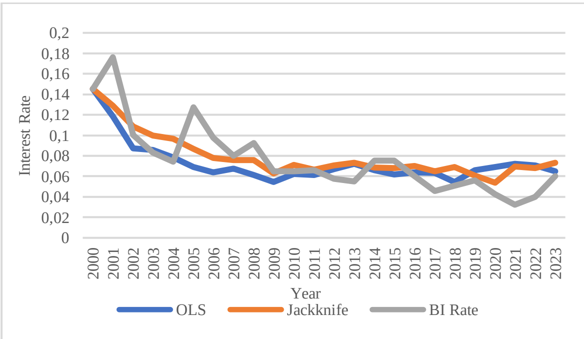

Figure 1. Hull-White Interest Rate for Each Parameter Estimation and BI-Rate Interest Rate

Figure 1 presents a comparison between the observed BI Rate and the interest rate paths generated by the Hull–White model under OLS- and Jackknife-based parameter estimation. All three series exhibit a clear downward trend over the observation period, indicating that both estimation methods are able to capture the mean-reverting behavior of interest rates. This result confirms the suitability of the Hull–White framework in modeling the long-term dynamics of policy interest rates.

Relative to the observed BI Rate, the Hull–White estimated interest rates display smoother trajectories, which reflect the stochastic structure of the model and the effect of mean reversion. A systematic difference is observed between the two estimation approaches, with the Jackknife-based estimates consistently lying slightly above those obtained using OLS. This pattern suggests that the Jackknife method produces parameter estimates that are less influenced by extreme observations, thereby yielding greater robustness to data variability.

The visual comparison is supported by the MAPE results, which show that the Jackknife method achieves a lower prediction error (24.4%) compared to the OLS method (26.1%). This indicates that the Jackknife-based Hull–White model provides superior predictive accuracy relative to OLS. Although the difference in MAPE values may appear modest, it is actuarially meaningful, as even small differences in estimated interest rates can lead to substantial changes in discount factors. Consequently, these differences have direct implications for the valuation of actuarial liabilities and the assessment of funding adequacy in pension and long-term benefit schemes.

Interest Rate Estimation of Hull-White Model

The interest rate estimation is conducted over 111 annual periods using the Hull–White one-factor model, with an initial interest rate set at 6%. Parameter estimates obtained from the Ordinary Least Squares (OLS) and Jackknife methods are used to generate projected short-rate paths, which are presented in Figure 2. This long projection horizon is intended to capture the long-term dynamics of interest rates relevant for actuarial valuation and funding analysis.

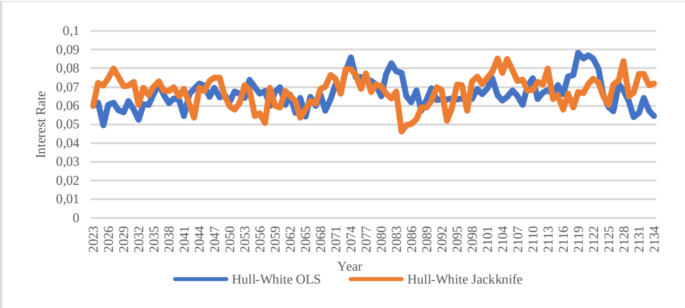

Figure 2. Hull-White Model Estimation

Figure 2 illustrates that the projected interest rate paths generated by both estimation methods exhibit similar overall trends, indicating consistency in capturing the mean-reverting behavior inherent in the Hull–White model. The simulated interest rates fluctuate around a stable long-term level, reflecting the balance between the mean reversion mechanism and stochastic volatility. This result confirms that both OLS and Jackknife estimators yield coherent representations of interest rate dynamics over extended periods.

Despite the similarity in overall patterns, noticeable differences emerge in the level and variability of the projected interest rates. In most periods, the interest rates estimated using the Jackknife method tend to lie above those obtained from OLS. This systematic difference is also reflected in the average projected interest rates, where the Jackknife-based estimates produce a higher mean value (0.0678) compared to the OLS-based estimates (0.0665). The relatively higher interest rate levels generated by the Jackknife method suggest that it is less influenced by extreme observations in the original dataset and therefore provides greater robustness to data variability.

From an actuarial perspective, these differences are economically meaningful. Higher projected interest rates lead to lower discount factors, which in turn reduce the present value of future benefit obligations. Consequently, the use of Jackknife-based parameter estimates may result in lower actuarial liabilities and improved funding indicators compared to those derived from OLS estimates. Therefore, although the numerical differences between the two estimation methods appear modest, their implications for long-term actuarial valuation and funding adequacy can be substantial.

Actuarial Calculation for the Frozen Initial Liability Method

The Hull–White interest rate model is applied in actuarial valuation using the Frozen Initial Liability (FIL) method, which assumes that benefit accruals are fixed at the valuation date and no additional service credits are accumulated thereafter. The initial monthly lifetime annuity is first calculated using commutation functions that incorporate survival probabilities and discount factors derived from the Hull–White estimated interest rate paths. The valuation is conducted for retirement ages of 58 and 60 years using parameter estimates obtained from both the Ordinary Least Squares (OLS) and Jackknife methods, with the resulting monthly annuity values presented in Table 4

Table 4. Monthly Annuity

Age	Annuity Using OLS	Annuity using Jackknife
58	19,0010	16,9269
60	22,6211	18,6175

The results indicate that the annuity values computed using Jackknife-based interest rate estimates are consistently lower than those obtained under OLS for both retirement ages. This outcome reflects the relatively higher interest rates generated by the Jackknife method, which produce stronger discounting effects and thus reduce the present value of annuity payments. The pension benefit calculation also accounts for the accumulated salary up to retirement, assuming an annual salary growth rate of 3%, with cumulative growth naturally slowing as retirement approaches. Overall, these findings highlight the sensitivity of actuarial liabilities to interest rate assumptions and demonstrate that the choice of estimation method within the Hull–White framework has a material impact on annuity valuation under the Frozen Initial Liability approac

a. Normal Contribution and Actuarial Liability

The calculation of normal contributions under the Frozen Initial Liability (FIL) method is influenced by both the contribution amount per participant and the number of active participants in each period. Under the FIL framework, the normal contribution per active participant remains constant from the time an individual joins the pension scheme until retirement, reflecting the assumption that benefit accruals are fixed at the valuation date. Consequently, changes in total normal contributions over time are primarily driven by demographic factors, particularly participant entry and exit from the program, rather than by adjustments in contribution rates. In this study, normal contributions are calculated based on interest rates generated by the Hull–White model using both Ordinary Least Squares (OLS) and Jackknife parameter estimation methods.

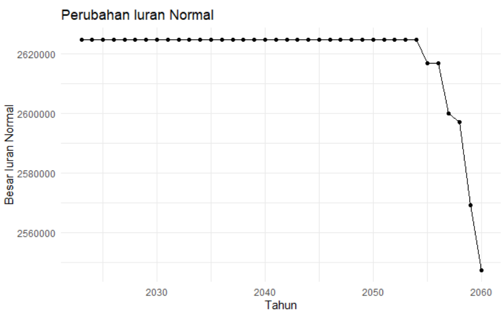

Figure 3a. Normal Contribution using OLS Method

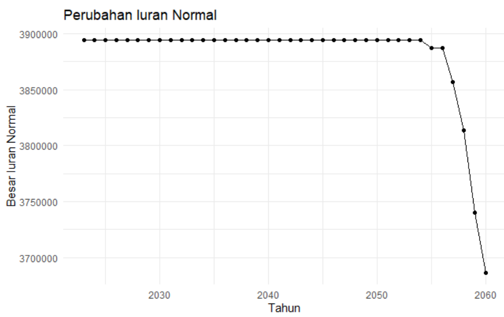

Figure 3b. Normal Contribution using Jackknife Method

Figures 3a and 3b illustrate the evolution of normal contributions derived from Hull–White interest rates estimated using OLS and Jackknife methods, respectively. In both figures, normal contributions remain stable over an extended initial period, consistent with the core assumption of the FIL method for active participants. A sharp decline is observed in later periods as participants approach retirement age, which leads to a reduction in the number of active contributors rather than a change in the contribution structure itself. While the overall patterns produced by both estimation methods are similar, the Jackknife-based normal contributions are consistently slightly lower than those obtained using OLS. This difference reflects the higher interest rates estimated under the Jackknife method, which result in stronger discounting effects and thus lower required normal contributions. These findings highlight the sensitivity of contribution requirements to interest rate assumptions and underscore the importance of robust interest rate estimation in actuarial funding analysis.

The normal contribution amounts estimated in the previous section serve as the basis for determining the actuarial liabilities that must be reserved by the pension fund. Using these contributions, actuarial liabilities are calculated over a projection horizon of 37 periods under the Frozen Initial Liability (FIL) method, with discount rates generated from the Hull–White interest rate model. Figures 4a and 4b present the projected actuarial liabilities based on interest rate estimates obtained using the Ordinary Least Squares (OLS) and Jackknife methods, respectively.

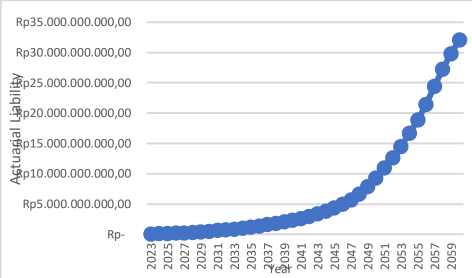

Figure 4a. Actuarial Liability Using OLS Method

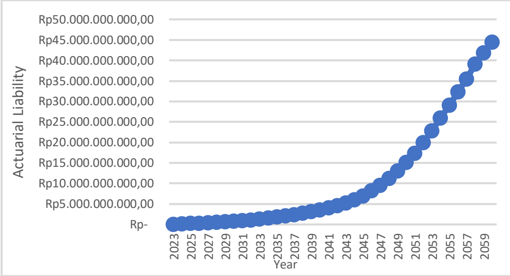

Figure 4b. Actuarial Liability Using Jackknife Method

Both figures display a clear exponential growth pattern in actuarial liabilities over time. In the early periods, liabilities remain relatively low, reflecting the long remaining time to retirement and the stronger impact of discounting. As participants approach retirement age, actuarial liabilities increase substantially due to the accumulation of pension benefits and the diminishing effect of discounting over shorter time horizons. Although the overall trajectories of actuarial liabilities under the OLS and Jackknife methods are similar, the liabilities estimated using the Jackknife method are consistently lower than those obtained using OLS. This outcome is consistent with the higher interest rate levels generated by the Jackknife-based Hull–White model, which lead to lower present values of future pension obligations. These findings underscore the high sensitivity of actuarial liabilities to interest rate assumptions and highlight the importance of robust interest rate estimation in pension funding and reserving analysis.

b. *Unfunded Actuarial Liability*

The computation of Unfunded Actuarial Liability (UAL) is essential for evaluating whether a pension fund possesses sufficient resources to fulfill its long-term actuarial commitments. UAL captures the funding position of a pension scheme by comparing actuarial liabilities with the accumulation of contributions and investment returns, thereby identifying potential funding gaps or surpluses. In this study, UAL outcomes are illustrated graphically to facilitate a clearer comparison between estimation approaches.

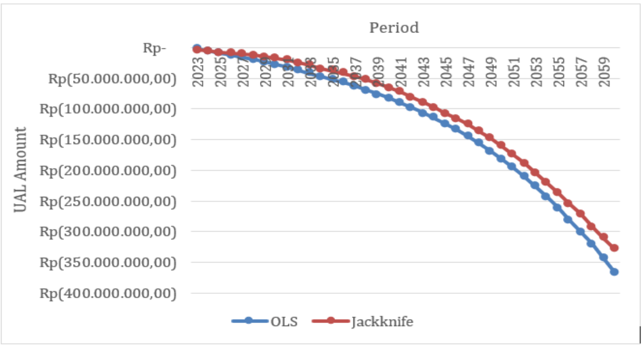

Figure 5. UAL Amount Using OLS Method and Jackknife

Figure 5 depicts the evolution of UAL over a 37-period projection horizon calculated under the Frozen Initial Liability (FIL) method, using interest rates generated by the Hull–White model with parameters estimated via Ordinary Least Squares (OLS) and Jackknife methods. Both curves exhibit a clear and persistent downward trajectory, with UAL values becoming progressively more negative as time advances. This pattern indicates that actuarial liabilities remain fully funded throughout the projection period, and that the pension scheme consistently operates in a surplus position rather than experiencing funding deficits.

A closer examination of the figure reveals systematic differences between the two estimation methods. The UAL series derived from the OLS-based Hull–White model lies below that obtained using the Jackknife method across most periods, implying that the OLS approach produces more negative UAL values and therefore a larger funding surplus. The divergence between the two curves gradually widens over time, reflecting the cumulative effect of differences in interest rate estimates on discounting future pension obligations. These differences arise primarily from variations in the estimated interest rate paths. The Jackknife method yields relatively higher interest rates compared to OLS, which results in stronger discounting of future liabilities and consequently lower present values of actuarial obligations. As a result, although both methods indicate a surplus position, the magnitude of the surplus is smaller under the Jackknife approach. Overall, Figure 5 highlights the sensitivity of pension funding outcomes to interest rate estimation methods and underscores the importance of robust interest rate modeling in assessing actuarial funding adequacy and long-term pension sustainability.

CONCLUSION

This study provides empirical evidence on the implications of interest rate estimation methods within the Hull–White framework for actuarial valuation. The results demonstrate that the Jackknife-based parameter estimation produces a slightly higher average interest rate of 0.0678 compared to 0.0665 obtained using the Ordinary Least Squares (OLS) method. More importantly, the Jackknife approach yields a lower prediction error, as reflected by a Mean Absolute Percentage Error (MAPE) of 24.4%, compared to 26.1% under OLS. These findings indicate that the Jackknife method enhances estimation robustness by reducing sensitivity to individual observations, which is particularly relevant given the relatively small sample size of the interest rate data used in this study.

When the estimated interest rates are applied to actuarial calculations under the Frozen Initial Liability (FIL) method, both estimation approaches result in negative Unfunded Actuarial Liability (UAL) values over the projection horizon, indicating that actuarial liabilities are fully funded and that the pension scheme operates in a surplus position. However, the magnitude of the surplus differs across methods. Specifically, the UAL values derived from the Jackknife-based Hull–White model are consistently less negative than those obtained using OLS, implying a smaller surplus despite the higher estimated interest rates. This outcome confirms the inverse relationship between discount rates and actuarial surplus, whereby higher interest rates reduce the present value of future pension obligations and consequently lower the measured surplus.

The novelty of this study lies in the integration of a robustness-oriented resampling technique, namely the Jackknife method, into the parameter estimation of the Hull–White interest rate model and its direct application to actuarial valuation using the Frozen Initial Liability approach. Unlike prior studies that rely predominantly on conventional OLS estimation, this research demonstrates that even modest improvements in interest rate estimation accuracy can lead to materially different actuarial outcomes, particularly in terms of annuity values, normal contributions, and funding surpluses. By explicitly linking interest rate estimation robustness to actuarial funding indicators, this study contributes new insights to the actuarial literature and underscores the importance of methodological choices in interest rate modeling for pension funding adequacy and actuarial decision-making.

REFERENCES

Ahmad, A. S., Alhusain, A. S., & Silalahi, S. A. (2015). Kebijakan Pengelolaan Dana Pensiun Sektor Korporasi. *Jurnal Ekonomi dan Kebijakan Publik, 06*, 78.

Alfikri, Z., Satyahadewi, N., & Perdana, H. (2020). Penggunaan Metode Frozen Initial Liability Pada Perhitungan Pendanaan Pensiun Manfaat Pasti (Studi Kasus: Data Pegawai Negeri Sipil di Puskesmas Kecamatan Hulu Gurung Kabupaten Kapuas Hulu). *Jurnal Buletin Ilmiah Mat. Stat. dan Terapannya, 09*(1), 47-56.

Andriananda, S. R., & Maulana, D. A. (2023). Kajian Metode Entry Age Normal Dan Projected Unit Credit Untuk Menghitung Kewajiban Aktuaria Pegawai Pemerintah Dengan Perjanjian Kerja. *Jurnal Ilmiah Matematika, 11*(03).

Caraka, R. E. (2016). Kajian Perhitungan Dana Pensiun Menggunakan Accrued Benefit Cost. *JURNAL BPPK, 9*, 160-180.

Choliqa, L. S. (2022). *Estimasi Parameter Suku Bunga Hull-White Menggunakan Metode Jackknife Dan Ordinary Least Square.* Malang: Universitas Islam Negeri Maulana Malik Ibrahim.

Gajek, L., & Ostaszewski, K. M. (2004). *Financial Risk Management for Pension Plans.* Amsterdam: Elsevier.

Gujarati, D. N., & Porter, D. C. (2009). *Basic Econometrics* (5th ed.). New York: McGraw-Hill.

Hull, J. C. (2009). *Options, Futures, And Other Derivatives.* New Jersey: Pearson.

Lesmana, D. (2015). Kecerdasan Spiritual Dengan Kecemasan Menghadapi Masa Pensiun. *Jurnal Ilmiah Psikologi Terapan, 02*, 169.

Lestari, S. D., & Mahrani, D. (2024). *Analisis cadangan Zillmer dana pensiun menggunakan metode aggregate cost pada tingkat suku bunga konstan dan stokastik*. Prosiding Seminar Nasional Sains Data, 4(1), 397–410

Maricar, M. A. (2019). Analisa Perbandingan Nilai Akurasi Moving Average dan Exponential Smoothing untuk Sistem Peramalan Pendapatan pada Perusahaan XYZ. *Jurnal Sistem dan Informatika, 13*(2).

Marwa, M. H. (2020). Analisis Status Badan Hukum Dana Pensiun. *Jurnal Yustika, 23*.

Pangestu, I. W., & Mahrani, D. (2023). Analisis Besar Iuran Normal Metode Frozen Initial Liability dan Metode Entry Age Normal Menggunakan Tingkat Suku Bunga Cox-Ingersoll Ross (CIR). *Indonesian Journal of Applied Mathematics*, 29-39.

Siahaan, M. M., Mahrani, D., & Sofia, A. (2024). *Analisis perbandingan cadangan premi New Jersey dan Canadian status joint life dengan model suku bunga Hull–White*. Indonesian Journal of Applied Mathematics, 4(2), 16.

Sprent, P. (1989). *Applied Nonparametric Statistical Methods.* New York: Chapman And Hall.

Taylor, H. M., & Karlin, S. (1998). An introduction to stochastic modeling. Academic Press.

Utami, A. H., Wilandari, Y., & Wuryandari, T. (2012). Penggunaan Metode Projected Unit Credit Dan Entry Age Normal Dalam Pembiayaan Pensiun. *JURNAL GAUSSIAN, 1*, 48-49.

Wiersema, U. F. (2008). *Brownian motion calculus*. John Wiley & Sons, Ltd.

Wooldridge, J. M. (2016). *Introductory Econometrics: A Modern Approach* (6th ed.). Boston: Cengage Learning.

Yunizar, E. D. (2019). *Estimasi Parameter Model Cox Ingersoll Ross Menggunakan Metode Jackknife* . Malang: Universitas Islam Negeri Maulana Malik Ibrahim.