INTERPOLTION OF SULFUR DIOXIDE (SO₂) AND NITROGEN DIOXIDE (NO₂) IN YOGYAKARTA SPECIAL PROVINCE USING THE COKRIGING METHOD

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Objective of this research are to determine which model is the best to use in interpolating the content of sulfur dioxide (SO₂) and nitrogen dioxide (NO₂) air pollution in the province of the Special Region of Yogyakarta using the kriging method. To Determine the interpolation of sulfur dioxide (SO₂).

1. Introduction

Air is a mixture of gases found in the layer that surrounds the Earth. The kriging method is the development of the kriging method to estimate a variable that minimizes estimation errors by utilizing cross-correlation between several variables. This research was conducted using data from 45 coordinate locations for air monitoring by the Department of Environment and Forestry of the Special Region of Yogyakarta in 2017. The results showed that the best model for estimating sulfur dioxide and nitrogen dioxide air pollution was the spherical model with the smallest Mean Square Error (MSE) value, which is 317,527. Interpolation of sulfur dioxide and nitrogen dioxide content values using cokring resulted in 100,390 new points. The value of air pollution is in the range of 0-40, which according to ISPU, means that at that point, good air quality has no impact on humans but causes a certain smell and injury to some plant species. This is an open-access article under the CC–BY-SA license.
2. Literature Review

2.1 Spatial Data

Spatial data is data obtained from measurement results that contain information about locations, generally based on maps that contain interpretations and projections of all phenomena that are on Earth [5].

2.2 Interpolation

Interpolation is a numerical procedure that forms a calculation of the height of a certain location based on the dependency function of the location surrounding it [6].

2.3 Kriging

\[ \hat{z}(u_0) = \sum_{i=1}^{n} w_i z(u_i) \]

2.4 Stationarity

The main requirement in analyzing geostatistical data using kriging is to fulfill the assumption of stationarity because it can minimize model errors. Data is said to be stationary if there are no drastic changes in the data, thus making the pattern random and not containing a trend.

\[ (Z_t) = \left( \frac{Z_t^\lambda - 1}{\lambda} \right), \quad \lambda \neq 0 \]

2.5 Experimental Semivariogram

An experimental variogram is the estimated value obtained from sampling and is made based on the spatial correlation value between two variables separated by a certain distance of \( y \). The experimental Variogram is formulated as follows [7]

\[ \hat{\gamma}(h) = \frac{1}{2|N(h)|} \sum_{i=1}^{N(h)} (z(s_i) - z(s_i + h))^2 \]

2.6 Experimental Semivariogram

After calculating the experimental semivariogram values, next is the calculation of the parameters used in the theoretical semivariograms. The following is an experimental semivariogram pattern:

![Fig 1. Experimental Semivariogram](image)

2.7 Theoretical Semivariogram

A theoretical variogram is a variogram that is arranged according to a particular function or model. Therefore, further analysis will be carried out in structural analysis, namely comparing the Mean Square Error (MSE) values between models on theoretical semivariograms. This aims to determine the best model, which is then used to assist the estimation calculation results. The following are three theoretical semivariogram models used as comparisons:

a. Exponential Model

The semivariogram form is formulated as follows:

\[ \hat{\gamma}(h) = \frac{C_0}{2\lambda} \sum_{i=1}^{N(h)} (z(s_i) - z(s_i + h))^2 \]
\[
\gamma(h) = c \left[ 1 - \exp \left( -\frac{h}{a} \right) \right]
\]

b. Spherical models

The form of this semivariogram is formulated as follows:

\[
\gamma(h) = \begin{cases} 
  c \left[ \frac{3}{2} \left( \frac{h}{a} \right) - \frac{1}{2} \left( \frac{h}{a} \right)^3 \right], & \text{jika } h \leq a \\
  c, & \text{jika } h > a 
\end{cases}
\]

c. Gaussian models

The form of this semivariogram is formulated as follows:

\[
\gamma(h) = c \left[ 1 - \exp \left( -\left( \frac{h}{a} \right)^2 \right) \right]
\]

d. Coriging

Coriging is an interpolation method that minimizes the variance of the estimation error by utilizing cross-correlation between several variables obtained from the main variable and covariate variables. Appropriate weighting will minimize the error variance, and the estimation results will be unbiased.[7]

\[
\hat{u}_0 = \sum_{i=1}^{n} a_i u_i + \sum_{j=1}^{m} b_j v_j
\]

e. Air Pollution

Environmental Pollution is the entry or inclusion of living things, energy substances, and/or other components into the environment by human activities so that the quality decreases to a certain level which causes the environment to not function according to its form [10]. The Table 1 shows the National Ambient Air Quality Standards. The ISPU value is used to categorize air quality conditions in a place. The following are air quality categories based on ISPU values.

<table>
<thead>
<tr>
<th>No</th>
<th>ISPU value</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-50</td>
<td>Good</td>
</tr>
<tr>
<td>2</td>
<td>51-100</td>
<td>Currently</td>
</tr>
<tr>
<td>3</td>
<td>101-199</td>
<td>Not healthy</td>
</tr>
<tr>
<td>4</td>
<td>200-299</td>
<td>Very Unhealthy</td>
</tr>
<tr>
<td>5</td>
<td>&gt;300</td>
<td>Dangerous</td>
</tr>
</tbody>
</table>

g. Sulfur Dioxide (SO\textsubscript{2})

Sulfur Dioxide (SO\textsubscript{2}) is a group of gases in the atmosphere with an unpleasant odor which is often found in industrial areas that use coal and cork as their main energy sources. Sulfur dioxide is a form of gas resulting from volcanic activity, volcanic eruptions, natural sulfur gas sources, hot springs, and natural hot steam [11].

h. Nitrogen Dioxide (NO\textsubscript{2})

Nitrogen Dioxide (NO and NO\textsubscript{2}, referred to together as NOx) is a highly reactive gas formed when oxygen and nitrogen react at high temperatures during combustion or in the event of a lightning strike [12].

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3. Research Methods
3.1 Location and Place of Research
   The research location was the Department of Environment and Forestry of the Special Province of Yogyakarta, and the place for the research was the Applied Statistics Laboratory, Statistics Study Program, Mathematics Department, Faculty of Mathematics and Natural Sciences, Tadulako University.

3.2 Population and Research Sample
   The population in this study is all air monitoring location points in the province of the Special Region of Yogyakarta. Meanwhile, the sample for this research is data from 45 coordinate locations for air monitoring by the Department of Environment and Forestry of the Special Province of Yogyakarta in 2017.

3.3 Data Collection Procedures
   The data used in this study is secondary data obtained from the results of the publication of the Yogyakarta Special Province Environment and Forestry Service in 2017. The variables used in this study are the coordinates in latitude (X), longitude (Y), Sulfur Dioxide (A) and Nitrogen Dioxide (B).

3.4 Data analysis
   a. Gather exploration data from the Department of Environment and Forestry of the Special Province of Yogyakarta
   b. Perform data stationarity testing. Stationary testing is carried out to see whether there is a trend in the data through whether the stationary assumptions are fulfilled or not. If the scatter plot on both data does not show a particular trend, then the spatial data meets the stationary assumption.
   c. Perform experimental semivariogram calculations and experimental cross-variograms for each variable.
   d. Calculating theoretical semivariogram and theoretical cross variogram using three models, namely spherical, gaussian, and exponential.
   e. Perform co-kriging interpolation using selected theoretical cross variograms and produce interpolation plots. Making conclusions from research results.

4. Results and Discussion
4.1 Data Exploration

![Plot of Sampled SO₂ Data](image)

Figure 2. Plot of Sampled SO₂ Data

Figure 2. shows the value of SO₂ air pollution content. From each coordinate point at 45 air monitoring points, where the blue color indicates a fairly large SO₂ air pollution content, the darker the blue color, the smaller the pollution content.
Figure 3. shows the value of NO$_2$ air pollution content. From each coordinate point, where the blue color indicates a fairly large NO$_2$ air pollution content, the closer to the darker blue, the smaller the pollution content. Furthermore, descriptive data from sampled air pollution points SO$_2$ and NO$_2$. Based on the calculation results shown in Table 3, it can be seen that the value of the minimum SO$_2$ air pollution content is 25.35 µg/$\cdot$m$^3$, which is located at one of the coordinate points (-7.836222222, 110.3446944), the minimum NO$_2$ air pollution content value is 23.86 µg/$\cdot$m$^3$ which is located at one of the coordinate point (-7.766277778, 110.3552222). The maximum SO$_2$ air pollution content value is 41.69 µg/$\cdot$m$^3$ located at coordinates (-7.836222222, 110.3743333), the maximum NO$_2$ air pollution content value is 36.05 µg/$\cdot$m$^3$ located at coordinates (-7.838472222, 110.41125), the average SO$_2$ air pollution content value is $m \space m^3 \space m^3 \space m^3 \space m^3 \space 34.136 \mu$g/$\cdot$m$^3$, the average value of NO$_2$ air pollution content is $m^3 \space 28.84 \mu$g/$\cdot$m$^3$ with the variance value for $m^3$SO$_2$ of 16,287 for NO$_2$ of 11,215, and the standard deviation value for SO$_2$ is 4,036, for NO$_2$ is 3,349.

4.2 Stationary Assumption

In kriging, it is necessary to assume that the data is stationary for the SO$_2$ and NO$_2$ pollution content variables. The stationarity of the data can be seen from the graph if it makes the pattern random, does not contain a trend, and if it appears to have a constant mean and variance value, as shown in Figure 4 and Figure 5 below:

![SO$_2$ sample data plot](image)

**Figure 4.** Plot of Sampled SO$_2$ Pollution Point Data

The plot above is a sample data plot of SO$_2$ air pollution points; from the plot above, it can be seen that the data is spread out on an average line, the pattern is random, and the line does not contain a particular trend, so it can be said that the data meets the assumption of stationarity.
Figure 5. Data Plot of Sampled NO₂ Pollution Points

Figure 5 is a sample data plot of NO₂ air pollution points; from the plot above, it can be seen that the data is spread on an average line, the pattern is random, and the line does not contain a particular trend, indicating that the data meets the assumption of stationarity and the next steps can be taken.

4.3 Experimental Semivariogram and Cross Variogram

The experimental semivariogram value is obtained based on the distance between the same points. Equation (2.7) The experimental semivariogram is symbolized by (gamma). This value can indicate a spatial correlation between observation points. Experimental semivariogram values obtained from each variable are experimental semivariogram values of SO₂, NO₂, and cross-variogram air pollution. Based on Table 4, there are 15 experimental semivariogram distances. Where the value is the number of adjacent data distance pairs, the value is the average distance based on the number of data pairs, and the value is the experimental semivariogram value calculated based on equation (2.7). In the first distance, the number of pairs of adjacent distances is obtained, namely 45 data distances with an average of 0.004, so based on the calculation results, the experimental semivariogram value is 19,678.

Figure 6. SO₂ Experimental Semivariogram Plot

From Figure 6, it can be seen that there are 15 distances to get the nugget, sill, and range values that will be used to calculate the theoretical semivariogram. The semivariogram value of NO₂ air pollution, the following is the resulting experimental semivariogram value. Based on Table 5, there are 15 experimental semivariogram distances. Where the value is the number of adjacent data distance pairs, the value is the average distance based on the number of data pairs, and the value is the experimental semivariogram value calculated based on equation (2.7).

Figure 7. Experimental Semivariogram Plot NO₂

Based on the NO₂ experimental semivariogram values in Table 5, when depicted in plot form, it will look like in Figure 7. The cross-variogram value is the observed value derived from two variables. Following are the cross-variogram values of the two variables, namely SO₂ air pollution and NO₂ air pollution. Based on Table 6, there are 15 experimental semivariogram distances. Where the value is the number of adjacent data distance pairs, the value is the average distance based on the number of data pairs, and the value is the experimental semivariogram value calculated based on equation (2.7).

Andina et al. (Interpolation Of Sulfur Dioxide (SO₂) And Nitrogen Dioxide (NO₂ ) In Yogyakarta Special Province Using The Cokriging Method)
4.4 Theoretical Semivariogram and Cross Variogram

The theoretical semivariogram and cross-variogram have the shape of the curve closest to the experimental semivariogram and cross-variogram model. In this study, there are three theoretical semivariogram, and cross-variogram models used, namely the Spherical, Gaussian, and Exponential models. The following is a theoretical SO2 semivariogram model.

**Figure 9. Theoretical SO2 semivariogram (a) Spherical Model, (b) Exponential Model, (c) Gaussian Model**

Figure 9. shows the three SO2 theoretical semivariogram models from the calculation results. The dot pattern indicates the experimental semivariogram model, while the lines indicate the theoretical semivariogram model. The spherical model has a nugget value of 0, a sill value of 0.276, and a range value of 0.006. The exponential model has a nugget value of 0, a sill value of 0.342, and a range value of 0.002, and the Gaussian model has a nugget value of 0, a sill value of 0.206, and a range value of 0.004. Furthermore, The following is a theoretical NO2 semivariogram model.
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Andina et al. (Interpolation Of Sulfur Dioxide (SO2) And Nitrogen Dioxide (NO2) Air Pollution Content Using The Cokriging Method)
degradation in the plot starts from orange, which indicates a very small point of air pollution value; from green to blue indicates the value of air pollution at a moderate point, while dark blue to purple indicates a high air pollution value. Figure 13 has air pollution values in the range of 0-40, according to the Air Pollution Standard Index (ISPU) value (Table 2) air quality at that point is good and has no impact on humans, but exposure to NO$_2$ in the range 0-40 causes a slight odor.

5. Conclusion

Based on the results and discussion that has been described previously, the conclusions that can be drawn are as follows:

1. The best model for interpolating the content of SO$_2$ and NO$_2$ air pollution in the Special Region of Yogyakarta using the cokringing method is spherical, with the smallest MSE value of 317,527.
2. Interpolation of SO$_2$ and NO$_2$ air pollution content using the coking method yields 100,390 new points from 45 starting points. Air quality prediction with color degradation in plots ranging from orange to purple indicates a very small level of pollution to a high level of air pollution. The SO$_2$ and NO$_2$ air pollution values at that point have a range of 0-40; according to the Air Pollution Standard Index (ISPU) value, the air quality at that point is good, with no impact on humans, but exposure to SO$_2$ with a range of 0-40 causes injury to several plant species and exposure to NO$_2$ in the range 0-40 causes a slight specific odor.

References

[1] The Republic of Indonesia Government Regulation Number 41 of 1999 concerning Air Pollution Control, Article 1 paragraph (4)

Appendix:

Table 1. National ambient air quality standards

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Andina et al. (Interpolation Of Sulfur Dioxide (SO\(_2\)) And Nitrogen Dioxide (NO\(_2\)) In Yogyakarta Special Province Using The Cokriging Method)

**Table 3.** Descriptive Data

<table>
<thead>
<tr>
<th>Variational</th>
<th>Min</th>
<th>Max</th>
<th>Means</th>
<th>Var</th>
<th>std. Dev</th>
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<td>SO(_2)</td>
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<td>41.69</td>
<td>34.14</td>
<td>16.29</td>
<td>4,036</td>
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<tr>
<td>NO(_2)</td>
<td>23.86</td>
<td>36.05</td>
<td>28.84</td>
<td>11.22</td>
<td>3,349</td>
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</table>

**Table 4.** SO\(_2\) Experimental Semivariogram Values

<table>
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<th>np</th>
<th>Dist</th>
<th>Gamma</th>
<th>id</th>
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<tbody>
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<td>0.004</td>
<td>19,678</td>
<td>SO(_2)</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>0.017</td>
<td>19,844</td>
<td>SO(_2)</td>
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<td>15</td>
<td>63</td>
<td>0.155</td>
<td>19,251</td>
<td>SO(_2)</td>
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</table>

**Table 5.** Experimental NO\(_2\) Semivariogram Values
Andina et al. (Interpolation Of Sulfur Dioxide (SO₂) And Nitrogen Dioxide (NO₂) In Yogyakarta Special Province Using The Cokriging Method)
Andina et al. (Interpolation Of Sulfur Dioxide (SO$_2$) And Nitrogen Dioxide (NO$_2$) In Yogyakarta Special Province Using The Cokriging Method)

<table>
<thead>
<tr>
<th>Point</th>
<th>Latitudes</th>
<th>Longitude</th>
<th>NO2</th>
<th>SO2</th>
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<tbody>
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<td>1</td>
<td>-7,783</td>
<td>110,412</td>
<td>31.14</td>
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